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AD849889

Technical Report No.138

SACLANT ASW
RESEARCH CENTRE

A SURVEY OF LITERATURE ON REFLECTION AND SCATTERING OF SOUND WAVES AT THE SEA SURFACE

by

Leonard FORTUIN

15 FEBRUARY 1969

NATO

VIALE SAN BARTOLOMEO, 400
I-19026-LA SPEZIA, ITALY

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TECHNICAL REPORT NO. 138

SACLANT ASW RESEARCH CENTRE
Viale San Bartolomeo, 400
I 19026 - La Spezia, Italy

A SURVEY OF LITERATURE ON REFLECTION AND SCATTERING OF SOUND WAVES AT THE SEA SURFACE

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Leonard Fortuin

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APPROVED FOR DISTRIBUTION
FOR THE DIRECTOR

R. Weller
Deputy Director

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ABSTRACT

Papers and reports published up to the middle of 1968 in the open literature are classified by subject and type of approach. They are analyzed, discussed and commented upon. General trends, relations between studies, agreements and contradictions are mentioned.

LIST OF SYMBOLS

A, A_m, A_{max}, A_{min}	Amplitude
a, a_x	Effective correlation distance
B, B_{AA}, $B_{\psi\psi}$	Correlation function
B_j	Boundary coefficient
b	Constant
C	Constant
c	Sound velocity
c_0	Sound velocity in ideal medium
D, D_x, D_y	Wave parameter
F	Surface wave spectrum
${}_mF_n$	Generalized hypergeometric function
f	Frequency of incident radiation (Hz)
G	Generalized spectrum
G_k	Green's function
g	Gravity acceleration (9.81 m/s^2)
H	Trough-to-crest surface wave height
H_n	Hankel function
h	Amplitude of sinusoidal surface: standard deviation of surface elevation
I, I_s, I_0	Intensity
I_n	Modified Bessel function

J	Autocovariance function of surface insonification
J_n	Bessel function
j	Integer
k	Wave number $(2\pi/\lambda)$ of incident radiation
L, L_x, L_y	Length of insonified area
ℓ	Distance; integer
M	Maximum number of scattering modes
m	Scattering mode number; integer
N	Size of statistical ensemble; integer
n	Surface normal; integer
O	Origin of coordinate system
P	Directivity pattern
p, p_o, p_1, p_T, p_r, p_s	Pressure
Q	Factor
q	Probability function
R	Receiver; shadowing function
R_o	Distance
r, r_{10}	Distance; radius
S	Surface; shadowing function
s	Surface profile; shadowing function
T	Transmitter
t	Time

U	Unit step function
u	Speed of surface wave; velocity potential
V	Reflection coefficient
v	Wind Speed
X	Operator
x, x'	Horizontal coordinate
y, y'	Horizontal coordinate
z, z_T, z_R	Vertical coordinate
α, α_T, α_R	Direction cosine; angle with X-axis
β, β_T, β_R	Direction cosine
Γ	Non-specular scattering coefficient
γ	Direction cosine
Δ	Layer thickness
δ	Dirac function
ϵ	Concentration of air bubbles
ζ	(normalized) Surface profile
ζ'	Surface slope
η	Difference in y-coordinates; normalized y-coordinate
θ, θ_m, θ_{in}, θ_{out}	Angle with vertical
K, K_x, K_y	Surface wave number ($2\pi/\Lambda$)
Λ	Surface period
Λ_M	Intensity of scattered waves

λ	Wavelength
λ, λ_o, λ_m	Direction cosine
μ	Index of refraction
μ, μ_o, μ_m	Direction cosine
ν	Direction cosine
ξ	Difference in x-coordinates; normalized x-coordinate
ρ, ρ_x, ρ_y, ρ_z	Correlation distance
ρ_o	Density
σ	Scattering coefficient
σ, σ_A, σ_ψ	(normalized) Standard deviation
σ_B	Backscattering strength
τ	Time difference
Φ, Φ_1, Φ_2	Surface correlation function
φ	Grazing angle
χ	Roughness parameter
Ψ	Correlation function of pressure
ψ	Phase angle
Ω	Spectral reflection coefficient
ω	Radial frequency of incident wave
ω_D	Frequency shift
ω_s	Frequency of surface wave

INTRODUCTION

Until modern times the oceans were of interest to man only as a source of food and as a medium that linked and separated the continents. The catching of fish, the transport of goods from one harbour to another, and the sea battles between warring nations, all these took place at the surface. Therefore little interest was shown in the ocean below the surface.

Recent times have seen the development of submarines, giving the war at sea one more dimension, and the increasing need of food for a growing world population, which makes more efficient fishing necessary.

Connected with this development is a diversity of technical systems that operate with underwater sound waves and are used for detecting an enemy (active and passive sonar), distinguishing friend from foe (IFF systems), tracing schools of fish, or measuring depth (fathometry).

There is at least one thing all these systems have in common: they can be considered as communication systems, since each one has a transmitter and a receiver, between which information is conveyed,

The medium that is used in these communication systems to carry the information from transmitter to receiver, i.e. the ocean, is certainly not perfect.

In the first place there is the phenomenon of a sound velocity changing with depth, that causes the formation of sound channels, caustics, shadow zones, etc.

Next there is the so-called volume reverberation, introduced by inhomogeneities in the medium (e.g. fluctuations in temperature, salinity, pressure, and small particles of biological nature), that influences the signals all along their propagation path and disturbs them in a random fashion.

Moreover, in many situations there is not only a direct path between transmitter and receiver, but also connection via the boundaries, especially at longer distances.

The signals that arrive at the receiver via these different paths may interfere or may be separated in time, depending on the geometry and the signal duration. If they interfere then one will probably try to build into the receiver a means of separating them. In the second case it is likely that the direct arrival will be given priority, as it carries the least disturbed information. Then the receiver will have to suppress the superfluous boundary-reflected signals, because their presence makes the system temporarily unusable for direct reception.

It is also possible to imagine a situation in which communication between transmitter and receiver can only take place via the bottom, or via the surface. This occurs when the receiver is positioned in the shadow zone of the transmitter.

From the above it can be concluded that it is essential for the designer of underwater communication systems to know how the propagation of sound is affected by the medium and its boundaries. A study of this effect can be split into three parts:

a) The volume.
b) The surface.
c) The bottom.

In this report we shall only be concerned with the surface effect.

As a first step in the study of "Reflection and Scattering of Sound Waves at the Sea Surface" it seems reasonable to investigate what work has been done in this field up to the present. The present literature survey is the result of this investigation.

1. GENERAL REMARKS

1.1 Definitions and Limitation of the Material

The problem of the diffraction of waves at uneven surfaces has received increasing attention in the past fifteen years; "this is due to the growing application of acoustic waves and radio waves in the centimetre band" (Ref. 45, p. 1).

Often diffraction is subdivided into "reflection" and "scattering", but these terms are not always distinguished clearly in the literature. In this work we shall call "reflection" that part of the diffracted field that travels in the specular direction (often named "specular reflection"). Waves in all other directions will be called "scattered waves" or simply "scattering". Scattering back towards the transmitter (backscattering) is also called "reverberation".

Mathematically the problem is "marvelously complex" (Ref. 76, p. 1293). It consists in solving a wave equation for which certain boundary conditions have to be satisfied, whereas the shape of the boundary can be extremely complicated. For this reason a general and exact treatment of the problem has not – so far – been published.

Nevertheless, a large number of publications in the open literature are devoted to the subject. But they only cover a part of the problem: all of them are restricted to a special case, and are based on certain assumptions – sometimes rather arbitrary – that make simplifications possible but at the same time cast doubt on their validity. Moreover they all deal with monochromatic waves.

The material can be limited if we consider the type of wave and the type of boundary. Both sound waves and electromagnetic waves (e.m. waves) give rise to the same type of mathematics, when reflection and scattering at uneven surfaces is studied. In fact, the mathematical formulation for sound waves can be considered as a simplified version of the one for e.m. waves, because for sound waves the vector equations are reduced to scalar equations. This is caused by the fact that sound waves do not possess the polarization that is inherent in e.m. waves.

Next, two types of boundary can be distinguished in practice, with some idealization:

a. The free, elastic boundary (e.g. the sea surface) on which the wave potential vanishes (homogeneous Dirichlet condition), i.e. the so called "pressure release" or "perfectly conducting" surface.

b. The rigid boundary (e.g. the rocky ocean floor) on which the first derivative of the wave potential becomes zero.

Except for the book by Beckmann and Spizzichino (Ref. 2), we shall only refer here to publications that deal with sound waves and perfectly reflecting, free boundaries; we do not, however, attempt to give a complete bibliography.

1.2 Sound Pressure and Velocity Potential

The terms "sound pressure" and "velocity potential" need some attention, as the way they are used in the literature may cause confusion.

The sound waves we are interested in are pressure waves: they can be described as a pressure field p that varies with time and position. Closely related to the field p is the wave velocity potential field u, as

$$p \approx \rho_0 \frac{\partial u}{\partial t}, \qquad \text{(Eq. 1)}$$

where ρ_0 is the mean density of the medium.

For monochromatic waves we have

$$u = |u|\, e^{-i\omega t} \qquad \text{(Eq. 2)}$$

so that Equation 1 reduces to

$$p \approx i\omega\rho_0 u. \qquad \text{(Eq. 3)}$$

It is this relation, only valid for monochromatic waves, that makes u and p interchangeable in the wave equation, in the boundary conditions, in the Helmholtz integral, and in all relations derived from them.

2. CLASSIFICATION OF THE LITERATURE

The diversity of special cases makes a classification of the existing literature rather difficult. However, an evaluation of the most pertinent material can be attempted by classifying each reference according to whether or not it treats of certain aspects of the subject. Such a classification is attempted in Table 1.* It is reviewed in Chapters 3 & 4 and conclusions are drawn in Chapter 5.

The division of the subject into different aspects is discussed in the remainder of the present chapter. The numbers and letters in Table 1 correspond to these divisions.

A. TYPE OF INCIDENT WAVE

A.1 Plane Waves

Directions appear instead of vectors, and all surface points are equally distant from the source. Considerable simplifications can be obtained, at the cost of loss of generality.

A.2 Spherical Waves

The source is of finite dimensions (in the limiting case a point source) at a finite distance from the boundary. This case is more realistic and more complicated.

B. TYPE OF SOURCE

B.1 Source with Directivity

Radiation takes place only inside a limited space angle, which restricts the active scattering area.

B.2 Omnidirectional Source

In experimental work this type of (point) source is usually obtained with explosives. In theoretical studies the active surface region becomes infinitely large, causing mathematical difficulties.

*pp. 20-22

C. DIMENSION OF THE MODEL

This criterion has only meaning for theoretical work.

C.1 Two-Dimensional Model

In many cases the analysis is limited to the "plane problem" (Ref. 45, p. 1). This means that the boundary is considered to be a function of only one space variable, so that the "surface" can be represented by a curve $z = \zeta(x)$. It is obvious that such models lack general validity: they are to be considered as a first step to gain insight.

C.2 Three-Dimensional Model

Especially in the case of point sources and point receivers it is highly desirable to represent the boundary by a function $z = \zeta(x, y)$. In principle the three-dimensional model can be obtained from the two-dimensional one, at the cost of more complicated expressions.

D. TYPE OF MODEL

The models used in the literature can all be characterized as "physical" models, with one exception: the "quasi-phenomeno-logical" approach of Middleton (Refs. 59, 60). In the physical models the inhomogeneity of the boundary is present in the formulation of the problem from the beginning, i.e. a solution of the wave equation is sought that satisfies certain boundary conditions. The phenomenological type assumes an ideal boundary and ideal wave propagation, and introduces the irregularities independently of the boundary as point scatterers with certain statistical properties. It is there that the difficulty of the method lies, for these properties are not easy to obtain. We shall therefore give most attention to the physical models.

An excellent survey of the models used up to 1958 has been given by Lysanov (Ref. 45). Although his paper deals only with periodically uneven surfaces, it has a wider importance, because many models can be applied to both periodically and statistically uneven surfaces. He described six methods of attacking the plane

problem, both for free and rigid boundaries, discussed their validity regions, and gave an extensive list of references from both Soviet and Western authors. The existence of a rather large number of theoretical models is due to the fact that the boundary conditions are difficult to incorporate in an exact way. This difficulty is caused by the complexity of the boundary. Some more or less arbitrary assumption has to be made in order to obtain a tractable approach. As for the assumption made, there are essentially two possibilities:

a. The diffracted field is assumed to have a certain structure (e.g. it is expandable in a series of plane waves: Rayleigh Method), after which the parameters are calculated via the boundary condition.

b. An assumption about the boundary condition is made, after which the field is calculated, mostly via the Helmholtz Integral (Ref. 1).

More or less parallel to this division runs the division into plane wave and spherical wave models. But this distinction is rather artificial, since it is possible to use a plane wave model, which only deals with directions, for the case of point sources and receivers, "by selecting the set of appropriate directions" (Ref. 11, p. 5).

The plane wave models use the methods D.1 to D.5. The spherical wave models all start with the well known Helmholtz Integral (Ref. 1) and then use either method D.6 or D.7 to approximate the first derivative of the field at the boundary needed to evaluate the integral.

D.1 The Method of Small Perturbations*

The boundary conditions on $z = \zeta(x)$ are transferred to $z = 0$ by means of a series expansion in ζ. The results are identical to those of D.3, when applied to periodic surfaces.

*From: Lysanov (Ref. 45).

D.2 Brekhovskikh's Method*

This approach is meant for relatively smooth surfaces that can be considered to be "locally flat". The amplitude of the irregularities may be large.

D.3 Rayleigh's Method

In this method the assumption is made that the scattered field can be represented everywhere by an infinite series of undamped plane waves.
The model was developed for a periodic boundary (see Section 3.2.1), but Marsh has generalized it for random surfaces (see Section 3.2.2). The validity of the basic assumption has been questioned by many authors (see Section 3.2.3), leading to improved versions of the Rayleigh method (e.g. D.4, D.5).

D.4 Variational Method

This method, developed by Meecham (Ref. 57), is an improvement of the Rayleigh approach. It calculates more accurately than Rayleigh the first N coefficients of the series, by an error-minimizing procedure.

D.5 Uretsky's Method

Being one of Rayleigh's critics, Uretsky has developed a modified version of the Rayleigh method, in which the wave equation is converted into an integral equation via a Green's function (Refs. 76, 77).
(See Section 3.2.4 for a summary of the method.)

D.6 Kirchhoff's Approximation

This method is also called "method of physical optics" (Ref. 2, p. 6). It is assumed that at the boundary the first derivative is equal for the incident wave and for the diffracted wave. This Kirchhoff approximation is somewhat arbitrary, as has been pointed out by

*From: Lysanov (Ref. 45).

Mintzer (Ref. 61), and is therefore also encountered in a modified form (Ref. 38). But in all cases the assumptions made leave the approximation open to discussion.

D.7 Integral Equation Method

The first derivative of the reflected field at the boundary is estimated via a Fourier integral, which is obtained assuming a receiver at the boundary. The method is therefore also called Fourier Transform Method; it is introduced by Meecham (Ref. 58) and described independently by Lysanov in his dissertation (see Ref. 45, p. 4).

* * *

D.8 Other Models

A model based on a different philosophy was prepared by Middleton (Refs. 59, 60). Instead of the classical or "physical" approach (as used in D.1 - D.7), he used a "quasi-phenomenological" approach in which the surface roughness is introduced as a random distribution of point scatterers on a perfectly flat boundary. (See Section 3.4 for details.) Still other models that do not fit in the foregoing scheme can be found in Beckmann's book (Ref. 2). Some of them are non-Kirchhoff methods.

E. TYPE OF SURFACE

Three types of boundaries can be distinguished, ranging from a poor approximation of the true ocean surface to a more realistic one:

E.1 Periodic Surfaces with Deterministic Profile

This type of boundary can be described exactly without invoking probability theory. A rigorous treatment of the problem is possible, mostly involving the (Rayleigh) expansion of the reflected field into an infinite set of plane waves.

E.2 Periodic Surfaces with Random Profile

For this type, probability theory is needed. The spatial correlation function of the surface elevation is periodic.

E.3 Random Surfaces

In this case the surface elevation and the slopes are considered to be stationary Gaussian processes. This is done primarily because only then can the analysis be continued up to a level where some conclusions can be drawn. Fortunately measurements at sea of elevation and slope have shown that the assumption of a "Gaussian sea" is satisfactory in most cases (Refs. 4, 24).

F. TIME

F.1 Time-Independent Surfaces

The larger part of the papers assume for simplicity a surface that does not depend on time.

F.2 Time-Variant Surfaces

More realistic is a surface of the type $z = \zeta\ (x, y; t)$. Then phenomena like Doppler-effect and frequency smear can be studied.

G. RELATIVE ROUGHNESS

Only in papers of a very theoretical character is there no statement about the relative size of the irregularities with respect to the wave length of the incident radiation. In others a "roughness parameter" appears, very often formulated via the Rayleigh criterion of roughness. This roughness parameter χ is proportional to the ratio h/λ, where h is the surface amplitude or the standard deviation of the surface elevation and λ the wavelength of the incident radiation. Then one or both of the following possibilities are considered:

G.1 Rough Boundaries

If the correlation between the elevation of neighbouring surface points is low, the surface is relatively very rough. Shadowing can occur at lower grazing angles. Scattering is diffuse.

G.2 Smooth Boundaries

Surfaces with good correlation are relatively smooth. Specular reflection is dominant.

H. THE SUB-SURFACE LAYER

Since the sea surface is the interface between air and water, both "elements" can mingle to a certain extent under favourable wind conditions. In this case the sub-surface layer contains a large number of small air bubbles that can produce a kind of volume scattering. In many cases the effect of this on surface scattering can be neglected; in certain cases, however (high wind speed, small grazing angles), the volume effect can screen the surface effect.

H.1 Ideal Layer

It is assumed that only the boundary causes the scattering and reflection and that the ocean itself is ideal everywhere.

H.2 Inhomogeneous Layer

In theoretical work this type is discussed by Lysanov (Refs. 46, 47). Many experimenters assume its existence in their explanation of data. The presence of air bubbles below the surface up to a certain depth, or a layer in which the sound velocity increases linearly with depth, is assumed.

I. THEORY AND EXPERIMENT

I.1 Theoretical Studies

The papers in this group are of a purely theoretical character, i.e. without experimental verification of the results obtained.

I.2 Experimental Work

In this group the results of experiments, mostly carried out at sea, are presented.

I.3 Comparison

Several papers start with a theoretical model, which is followed by a comparison with own data or with data from other publications. Experimenters may also "borrow" a theoretical model for comparison.

J. MAIN SUBJECT OF THE PUBLICATION

Almost all publications assume the characteristics of the surface to be known, i.e. they describe the surface with a deterministic and periodic function, or presuppose the statistical properties of the boundary. These publications deal with the following subjects and quantities.

J.1 Rigorous Solution of the Wave Equation

In papers of a very theoretical character Rayleigh's expansion of the field, diffracted at a periodic surface or at a random boundary (Marsh - Ref. 48) into an infinite series of plane waves, is adopted, with or without modification (Uretsky - Refs. 76, 77 and Meecham - Ref. 57) for the surface "valleys"; the amplitudes of the waves are calculated. In the case of a random surface this is done via Wiener's Generalized Harmonic Analysis.

J.2 Amplitudes of the Diffracted Field; Reflection Coefficients

Some model studies have been performed to check the above rigorous solutions for periodically uneven surfaces. Reflected and scattered amplitudes of order zero (= specular reflection) to

m (m = -1, -2, -3 ...) have been measured (backscattering). Sometimes the results are normalized to obtain reflection coefficients. Measurements at sea have also yielded a reflection coefficient (Refs. 42, 67).

J.3 Second Order Statistical Moments of the Diffracted Field

In this category the following subjects are encountered:

a) Reflected and scattered intensity.
b) Power reflection coefficient.
c) Scattering strength (forward, backward).
d) Scattering cross section.
e) Amplitude and phase fluctuations.
f) Spatial correlation of field amplitudes.

This is the largest group, containing both theoretical and experimental results. In many cases the dependency on grazing angle, frequency, or wind speed is investigated.

J.4 "Doppler" and other Frequency Effects

A small number of papers recognize the fact that the surface is time-variant. Then "Doppler effect" and "frequency smear" are studied.

J.5 Geometrical Shadowing

A special group of articles is devoted to the shadowing of surface "valleys" by neighbouring "peaks", which can occur at high frequencies and small grazing angles.

J.6 The Inverse Problem

This is the case when the parameters that characterize the surface are inferred from the properties of the diffracted field.

J.7 Sea Surface Wave Spectrum

The theory of a surface wave spectrum is discussed in papers of more recent date. This theory provides an estimate of the surface correlation function that is more realistic than the arbitrarily chosen functions in earlier work.

TABLE 1

CLASSIFICATION OF REFERENCES

Ref. no.	Author(s)	A	B	C	D	E	F	G	H	I	J
8	Abubakar, I.	1		1	1	1	1		1	1	1
9	Adlington, R.H.		2							2	3
10	Andreeva, I.B. et al.		2						2	2	3
11	AVCO Marine El. Off.	1,2		2	3	3	2	2	1	3	3,4 6,7
12	Batantsev, R.G.	1		2	3	1	1		1	1	1
13	Barnard, G.R. et al.	1		1	5	1	1	1	1	3	2
14	Beckmann, P.	1						1		1	5
15	Berman, A.			1	8	3				3	2
16	Brockelman, R.A. et al.	1						1		3	5
17	Brown, M.V. et al.		1							2	3
18	Brown, J.R. et al.		2							2	3
19	Chapman, R.P. et al.		2						2	2	3
20	Chapman, R.P. et al.		2							3	3
21	Chuprov, S.D.	2		2	1	3	2	2	1	1	3
22	Clay, C.S.	2	2	2	6	3	1	2	1	3	3,6
23	Clay, C.S. et al.	1		1	6	3	1	1	2	3	3
24	Cox, C. et al.									2	6
25	D'Antonio, R.A. et al.	2	1	2	8	3	2	2	1	3	2,3 4
26	Eckart, C.	2	1	2	6	3	1	2	1	1	3,6
27	Fante, R.L.	1		1	8	3	1	1,2	1	1	3
28	Garrison, G.R. et al.		1							2	3
29	Glotov, V.P. et al.	1		1	1	2	1	2	2	1	2
30	Gulin, E.P.	2	1	2	6	3	1	2	1	1	3
31	Gulin, E.P.	2	1	2	3,6	1	2	2	1	3	3
32	Gulin, E.P. et al.	2	1				1			2	3
33	Gulin. E.P.	2		2	1	2	1	2	1	1	3
34	Gulin. E.P. et al.	2						2		2	3
35	Hayre, H.S. et al.	1		2	6	3	1	1,2	1	1	3
36	Heaps, H.S.	1,2	2	2	3	1	2	1,2	1	1	2,3
37	Heaps, H.S.	1		2	3	1	1	2	1	3	2
38	Horton, C.W. et al.	2	1	2	6	2,3	1	2	1	1	3

Ref. no.	Author(s)	A	B	C	D	E	F	G	H	I	J
39	Horton, C.W. et al.	2	1	2	6	3	1	2	1	3	3
40	Kur'yanov, B.F.	1		2	1,6	3	1	2	1	1	3
41	La Casce, E.O. et al.	1,2	1	1	2,3,6	1	1	1,2	1	3	2
42	Liebermann, L.N.	2	1		8	3	2			2	2
43	Liebermann, L.N.									2	4,6
44	Lippmann, B.A.	1			3					1	1
45	Lysanov, Yu.P.	1		1	1,2,3 7,8	1 3				1	1,2
46	Lysanov, Yu.P.	1		2	3	1	1		2	1	1
47	Lysanov, Yu.P.	1		1	3	1	1	2	2	1	1,2
48	Marsh, H.W.	1		1	3	3	1		1	1	1,3
49	Marsh, H.W. et al.	1		2	3	3	2	2	1	1	3,7
50	Marsh, H.W.	1		2	3	3	2	2	1	1	3
51	Marsh, H.W.			2	3	3	2	2	1	3	3,7
52	Marsh, H.W.	1			3					1	1
53	Marsh, H.W.	1	1	1	3	3	2		1	1	4
54	Marsh, H.W. et al.				3,6 8					3	2,4 7
55	Martin, J.J.						2		2	3	3,7
56	Medwin, H.	2	1	2	6	3	2	1,2	1	3	3,6
57	Meecham, W.C.	1		1	4	1	1	2	1	1	2,3
58	Meecham, W.C.	1,2	2	1	7	1,2	1	2	1	3	2,3
59	Middleton, D.	1,2	1,2	2	8	1,2 3	2	1,2		1	2,3 4,5
60	Middleton, D.	1,2	1,2	2	8	1,2 3	2	1,2		1	2,3 4,5
61	Mintzer, D.	2	1	2	6		1	2	1	1	1
62	Murphy, S.R. et al.	1		1	3,5	1			1	1	1,2
63	Parker, J.G.	1		2	3	1	1	2	1	3	2
64	Parker, J.G.	1		2	3	1	1	2	1	3	2
65	Parkins, B.E.	1		2	6	3	2	2	1	3	3 4,7
66	Patterson, R.B.			1	8	3	1		1	1	3
67	Pollak, M.J.	2	1							2	2
68	Proud, J.M. et al.	1		1	4	1	1	2	1	3	2
69	Proud, J.M. et al.	2	1	1	6	3	1	1,2	1	3	3,6
70	Richter, R.M.	2	2							2	3

Ref. no.	Author(s)	A	B	C	D	E	F	G	H	I	J
71	Rojas, R.R.					3	2			1	4
72	Schulkin, M. et al.						2	1,2		3	3,7
73	Shaw, L.	1						1		1	5
74	Smirnov, G.E. et al.	2	1	2		1,3		1,2	1	2	3
75	Smith, B.G.	1						1		3	5
76	Uretsky, J.L.	1		1	5	1	1	1,2	1	3	2
77	Uretsky, J.L.	1		1	5	1	1	1,2	1	1	1
78	Urick, R.J.	2	1						2	2	3
79	Urick, R.J. et al.	2	1						2	2	3
80	Wagner, R.J.	1						1		3	5

3. COMMENTS ON THE LITERATURE

3.1 Introduction

The phenomenon of scattering and reflection of sound waves at the sea surface, generally speaking, takes place simultaneously in three domains:

a. Time

The ocean surface is continuously in movement, due to winds and currents. A realistic description of this surface and its reflection properties is therefore impossible without involving the time variable. Most promising seems the Neumann-Pierson model of ocean waves, based on a surface whose elevation and slopes can be considered as stationary Gaussian processes. This subject is discussed in greater detail in Section 4.8.

b. Frequency

The scattering and reflection properties of the surface are not only a function of time, but also of the signal frequency. For very high frequencies a behaviour similar to "geometrical optics" is likely: shadowing of "valleys" by "peaks" may occur (see Section 4.6), whereas for low frequencies the waves will be diffracted and reach all surface points.

c. Space

The diffracted field depends strongly on the relative position of source and receiver with respect to the boundary. The shadowing mentioned in (b) will become increasingly important when the grazing angle approaches 0. Volume-scattering due to an inhomogeneous sub-surface layer can also take place then.

A general statistical description of the diffracted field, complete up to second order statistical moments, therefore requires both a realistic surface model that takes into account the possibility of shadowing and sub-surface scattering, and observation of the field at two separately located receivers, at two frequencies, and at two instants of time. Only then one can obtain knowledge

about the following subjects:

Impulse response of the surface
Frequency spreading of signals due to the Doppler effect
(Coherence limits)
Curvature of the wave fronts.

Our first and general conclusion may therefore be that most of the papers analysed give a very incomplete description of the scattering and reflection of sound waves by the ocean surface, as they deal — roughly speaking — only with the following features.

a. Time-Independent Surfaces

The sinusoidal boundary is often encountered (Refs. 13, 31, 36, 37, 64, 76), and the saw-tooth also occurs (Ref. 68). Both of them are very rough approximations of the true sea boundary.

The random surfaces are based on the assumption of a stationary Gaussian process, mainly for computational reasons. Analysis of the sea surface has shown that this assumption is not far from the truth (Refs. 4, 24). The spatial correlation function of the surface, however, is often arbitrarily chosen, e.g. exponential or Gaussian, again with the excuse that it makes continuation of the calculations possible. In more recent publications the Neumann-Pierson model of ocean wave spectra is receiving increasing attention (Refs. 49, 51, 55, 65, 72).
An intermediate position is taken by the random periodic surfaces (Refs. 21, 29, 30, 33).

b. Monochromatic Waves

Sometimes, in experimental work, a pulsed-CW source is used (Refs. 17, 28, 32, 34, 39, 56, 67, 68, 69), or even explosives (Refs. 9, 10, 18, 19, 20, 51, 70), but then the analysis is done via narrow-band filters, reducing it to the monochromatic case again.

c. One Receiver

Exceptions are found in the Russian literature (Refs. 33, 34).
(See Section 4.2)

d. No Shadowing

This subject is treated separately (Refs. 14, 16, 73, 75, 80). (See Section 4.6)

e. Ideal Sub-Surface Layer

In experimental work the presence of such a layer is sometimes hypothesized (Refs. 10, 19, 23, 78, 79). Russian authors have investigated its influence in some theoretical work (Refs. 29, 46, 47). (See Section 4.4)

There is one important exception to this general conclusion: the quasi-phenomenological approach of Middleton (Refs. 59, 60). A very short description of this approach can be found in Section 3.4.

3.2 The Rayleigh Method and Related Solutions of the Wave Equation with Boundary Condition

At the end of the 19th century Lord Rayleigh studied the scattering of sound waves at periodically corrugated surfaces (Ref. 6). His method can be considered as the first attempt to solve the wave equation in combination with a boundary condition. It is an intuitive approach that has been used by many investigators, often with modifications, up to the present day. The Rayleigh method is described in Section 3.2.1.

The periodicity of the boundary prompted Rayleigh to expand the reflected field into a set of undamped plane waves. His assumption that this expansion is valid up to the boundary (which he made to use the boundary condition) has been questioned by many authors. "However, ... no rigorous proof of the invalidity of Rayleigh's method has ever been published" (Ref. 77, p. 402).

Although Rayleigh's method was originally suggested by periodicity of the boundary, it has been extended by Marsh to random surfaces. Details of this generalization can be found in Section 3.2.2.

3.2.1. Rayleigh's Method for a Sinusoidal Surface

A simple and straightforward description of the Rayleigh method for a periodic boundary is given by Beckmann (Ref. 2, Chapter 4), from which the following is a summary.

A plane monochromatic sound wave with wavelength λ is incident on an infinitely long periodic boundary with angle of incidence θ. In its most simple form such a boundary can be described by:

$$z = \zeta(x) = \zeta(x + \Lambda) \qquad (-\infty < x < \infty), \qquad \text{(Eq. 4)}$$

where Λ is the period of the surface corrugation. Because of the periodicity of the surface the diffracted field is assumed to propagate in certain discrete modes, making angles θ_m with the vertical that are given by the grating formula:

$$\sin\theta_m = \sin\theta + m\,\lambda/\Lambda \qquad (m = 0, \pm 1, \pm 2, \ldots),$$

or in terms of the wave numbers k and K

$$\sin\theta_m = \sin\theta + m\,K/k. \qquad \text{(Eq. 5)}$$

We remark that for $m = 0$ the reflection is "specular".

According to Eq. 5, θ_m can only assume discrete values when λ and Λ are held constant. These are the directions of scattering. They have the property that in these directions the waves scattered from individual periods reinforce each other because their phase difference is an integral number of periods.

For a sinusoidal surface, namely for

$$\zeta(x) = h\cos(Kx) \qquad (-\infty < x < \infty) \qquad \text{(Eq. 6)}$$

Rayleigh calculated the amplitudes A_m $(m = 0, \pm 1, \pm 2, \ldots)$ of the scattered waves via the boundary condition $p = 0$, where p is the total pressure field. His procedure for obtaining a solution of

the wave equation, i.e. the coefficients A_m, is based on two assumptions:

a. That the total field can be written as an infinite sum of plane waves:

$$p(x,z) = \exp\left[ik(x \sin\theta - z\cos\theta)\right] + \sum_{m=-\infty}^{\infty} A_m \exp\left[ik(x \sin\theta_m + z\cos\theta_m)\right]$$

(Eq. 7)

(the first term on the right hand side being the incident wave).

b. That this equation holds everywhere above and on the boundary. This assumption is not at all obvious and has been seriously criticized. (See Section 3.2.3)

With his two assumptions Rayleigh found that for a point (x,z) at the boundary

$$\exp\left[-ik\zeta(x)\cos\theta\right] = -\sum_{m=-\infty}^{\infty} A_m \cdot \exp\left[imKx + ik\zeta(x)\cos\theta_m\right]. \qquad \text{(Eq. 8)}$$

"Both sides of this equation are now expanded in a Fourier series with respect to x (which will in general result in a double series on the right side) and the resulting Fourier coefficients are equated. This results in an infinite set of linear equations, each of which contains the unknown coefficient A_m. By progressive solution (or successive approximation) the coefficients A_m are then approximated" (Ref. 2, p. 43). Formulae for the first coefficients can be found in Table 2.

The total number of possible modes as predicted by Eq. 5 is limited by the condition $|\sin\theta_m| \leq 1$. We call this maximum M. For $m > M$ the condition is violated. Then $\cos\theta_m$ becomes imaginary and we have (See Eq. 8) waves propagating along the surface (Rayleigh surface waves) that decay exponentially with depth.

The propagation in discrete modes described here is valid for "surfaces" that extend from $-\infty$ to $+\infty$. It is interesting to note what happens when the periodic surface is of finite length. Then the diffracted field — instead of being cancelled completely because of destructive interference between the directions given by the grating formula (Eq. 5)— decreases gradually and then increases again, when the observer is moved from the direction θ_m to $\theta_{m \pm 1}$. In this way the so-called "lobes" are formed. Their width increases as the surface becomes shorter.

For several combinations of θ, Λ and kh, Beckmann (Ref. 2) gives figures that illustrate this formation of lobes (Fig. 1 and 2). They show that with decreasing value of kh the "roughness" becomes smaller so that fewer and fewer sidelobes appear and the lobe with $m = 0$ (specular reflection) becomes more and more pronounced. With constant kh and Λ the reflection becomes more specular as θ increases. Both facts agree with a definition of roughness of the form

$$\chi = Ckh \cos \theta . \qquad \text{(Eq. 9)}$$

Other authors (Refs. 8, 12) considered an infinitely long periodical boundary, i.e. they studied the set of amplitudes A_o, $A_{\pm 1}$, $A_{\pm 2}, \ldots A_{\pm m}$. Abubakar (Ref. 8) arrived at some interesting conclusions:

a. If $kh \ll 1$, the non-specularly reflected waves are small, irrespective of Λ. Specular reflection is then dominant.

b. If $\Lambda \ll \lambda$, surface waves can occur. Part of the incident energy is then trapped in the "valleys", at the expense of the undamped scattered waves. These can become completely negligible, so that, if Λ is small enough, only specular reflection $(m = 0)$ remains. This agrees with Beckmann (Ref. 2, p. 36).

3.2.2 The Marsh-Rayleigh Method for a Random Surface

The method of Lord Rayleigh for a sinusoidal boundary has been generalized by Marsh for the case of a random surface (Ref. 43).

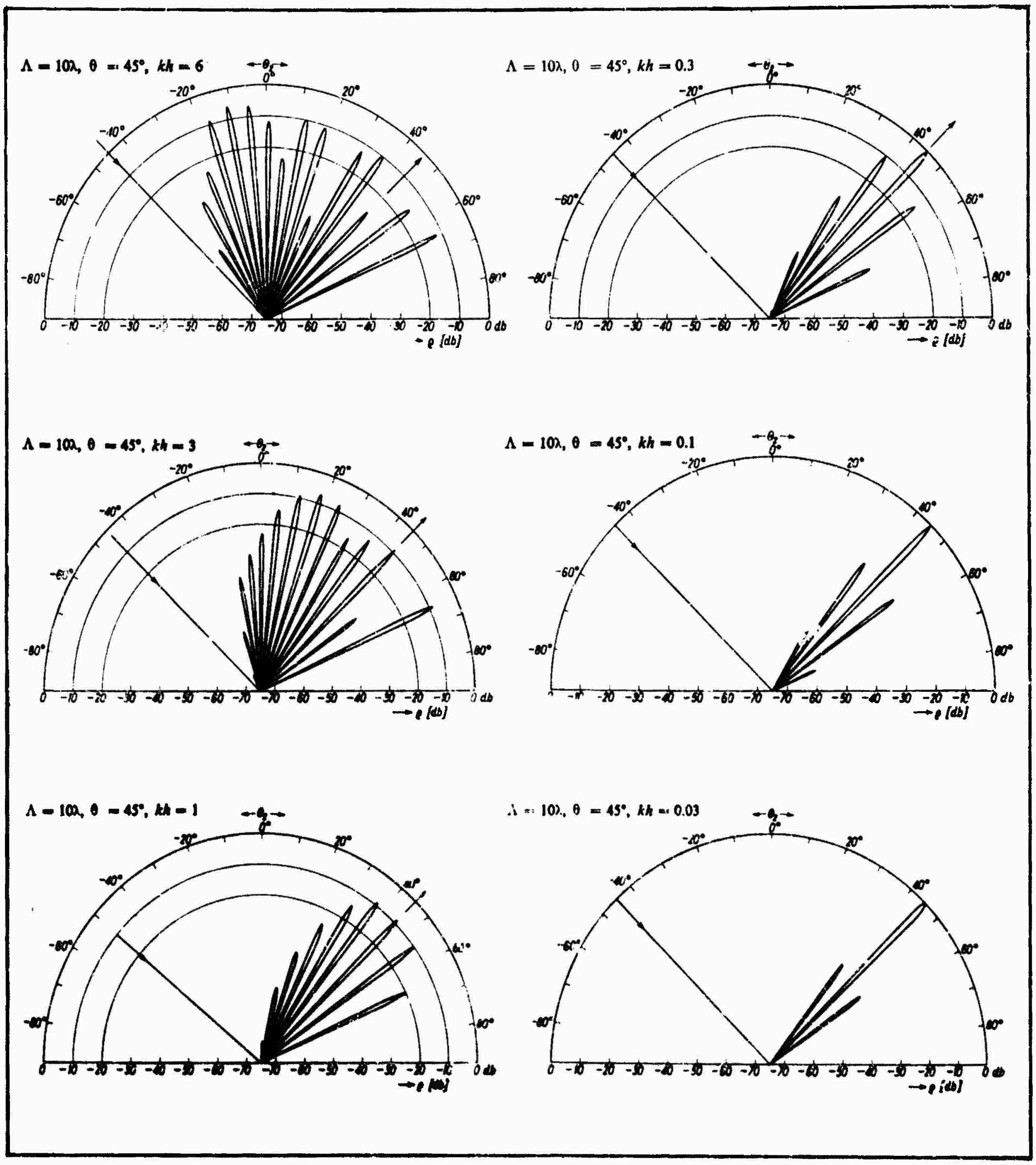

FIG. 1 DIFFRACTION OF A PLANE HARMONIC WAVE BY A SINUSOIDAL BOUNDARY OF FINITE LENGTH
Wavenumber $k = 2\pi/\lambda$; $\zeta(x) = h\cos(2\pi x/\Lambda)$; angle of incidence $\theta = 45°$, kh is a measure for the surface roughness. (From Beckmann and Spizzichino – Ref. 2, pp. 50–56)

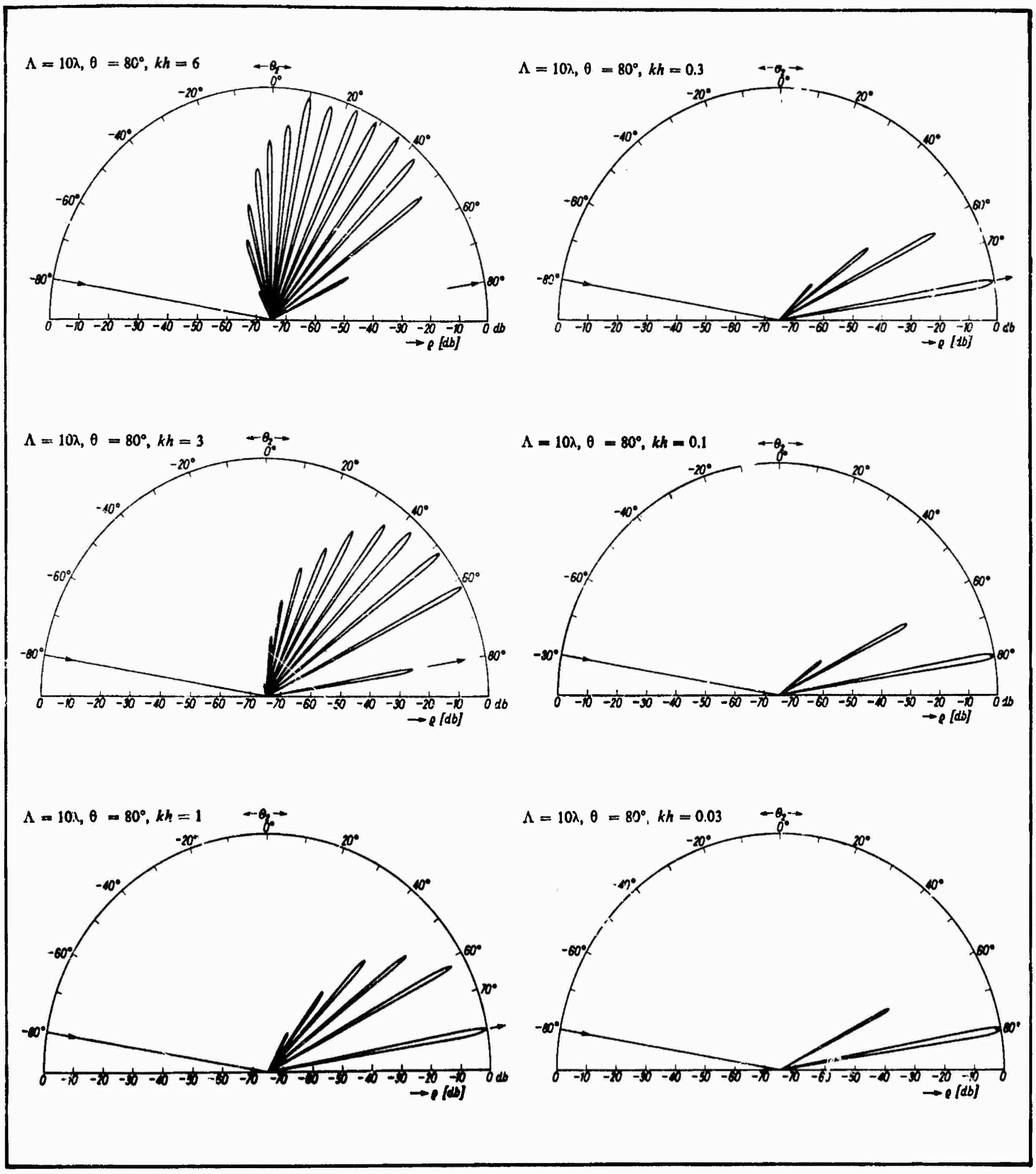

FIG. 2 DIFFRACTION OF A PLANE HARMONIC WAVE BY A SINUSOIDAL BOUNDARY OF FINITE LENGTH
Wavenumber $k = 2\pi/\lambda$; $\zeta(x) = h\cos(2\pi x/\Lambda)$; angle of incidence $\theta = 80°$, kh is a measure for the surface roughness. (From Beckmann and Spizzichino – Ref. 2, pp. 50–56)

He published his generalization "in an heuristic form, in order to avoid presenting the exceedingly heavy analysis required for a rigorous treatment" (p. 330). This omission of sufficient comments on the basic steps in his paper, together with a rather large number of misprints, makes his article somewhat hard to follow.

Marsh's extension of the Rayleigh method is obtained via Wiener's concept of "Generalized Harmonic Analysis" (Refs. 5, 7). It produces an expression for the correlation function of the scattered field at two points in space in a horizontal plane below the rough surface upon which a plane monochromatic sound wave is incident, but "this solution is readily extended to include electromagnetic waves, general elastic waves, and non-planar, non-harmonic sources" (p. 330 - abstract).

The "exact" solution for the problem of wave scattering by irregular surfaces can be summarized as follows.

A monochromatic plane wave (direction cosines α, β, γ) is incident on a random pressure relief boundary $S\left[z = s(x,y)\right]$. For the diffracted field $p_1(x,y,z)$ a plane wave representation is sought by writing

$$p_1(x,y,z) = \iint_{-\infty}^{+\infty} \exp\left[-ik(\lambda x + \mu y - \nu z)\right] dG(\lambda,\mu), \qquad \text{(Eq. 10)}$$

where $G(\lambda,\mu)$ is the generalized spectrum of $p_1(x,y,z)$ and λ, μ, ν are the direction cosines of the diffracted wave (hence: $\lambda^2 + \mu^2 + \nu^2 = 1$). The expansion (Eq. 10) is a straightforward generalization of the Rayleigh method for a periodic surface, in which $p_1(x,y,z)$ was decomposed into an infinite series of plane waves (See Eq. 7).

Rayleigh's second assumption, that the expansion is valid up to the boundary, is also adopted by Marsh; the criticisms of Rayleigh's approach apply therefore equally to Marsh (See Section 3.2.3).

With the boundary condition of zero total pressure and after normalization of variables: $kx = \xi$, $ky = \eta$, $ks(x,y) = \sigma\zeta(\xi,\eta)$, $\sigma^2 = k^2h^2$, $h^2 = \langle(s - \langle s\rangle)^2\rangle$ and $\langle(\zeta - \langle\zeta\rangle)^2\rangle = 1$, Marsh obtained

$$\exp\left[-i(\alpha\xi + \beta\eta + \gamma\sigma\zeta)\right] + \iint_{-\infty}^{+\infty} \exp\left[-i(\lambda\xi + \mu\eta - \nu\sigma\zeta)\right] dG(\lambda, \mu) = 0 . \quad \text{(Eq. 11)}$$

After this he expanded $G(\lambda, \mu)$ in a power series in σ:

$$G(\lambda, \mu) = \sum_{m=0}^{\infty} \sigma^m A_m (\lambda, \mu) \quad \text{(Eq. 12)}$$

and the coefficients A_m are to be calculated. Substitution of Eq. 12 into Eq. 11 yields an infinite set of simultaneous linear equations for the determination of the $A_m (\lambda, \mu)$. By clever manipulation of these equations Marsh found a simple-looking expression for the scattered field at a point not on the boundary. Choosing the coordinate system in such a way that the point of observation lies in the plane $z = 0$ (this includes: $\langle\zeta(x,y)\rangle \neq 0$, in contrast to most other theories) he obtained:

$$p(\xi, \eta, 0) = - \frac{\exp\left[-i(\alpha\xi + \beta\eta + \gamma\sigma\zeta)\right]}{1 + X} \quad \text{(Eq. 13)}$$

where X is a complicated operator closely related to the basic expression in Wiener's work.

Marsh, Schulkin and Kneale (Ref. 49) have worked out the method in more detail, assuming σ so small that $G(\lambda, \mu)$ can be represented satisfactorily with three terms of the series in Eq. 12. The necessary condition for this approximation was not discussed. They found that

$$d A_0 = -\delta(\lambda - \alpha)\ \delta(\mu - \beta)\ d\lambda d\mu$$

$$d A_1 = 2i\gamma F\left(\zeta e^{-i(\alpha\xi + \beta\eta)}\right) d\lambda d\mu \qquad \text{(Eq. 14)}$$

$$d A_2 = 2\gamma F\left(\zeta\right) * \left[\nu F\left(\zeta e^{-i(\alpha\xi + \beta\eta)}\right)\right] d\lambda d\mu$$

where $F\left(\cdot\right)$ denotes the Fourier transform and $*$ means convolution.

Equations 10, 12 and 14 with $z = 0$ gave an expression that approximates Eq. 13.
The correlation function

$$\Psi(\xi, \eta) = \langle p(\xi_1, \eta_1, 0)\ p^*(\xi_1 + \xi, \eta_1 + \eta, 0)\rangle \qquad \text{(Eq. 15)}$$

then followed easily:

$$\Psi(\xi, \eta) = e^{-i(\alpha\xi + \beta\eta)} \left[1 + 4\gamma^2\sigma^2\Phi(\xi, \eta) - 4\gamma\sigma^2 \iint_{-\infty}^{+\infty} \nu F(\lambda - \alpha, \mu - \beta)\, d\lambda d\mu\right]. \qquad \text{(Eq. 16)}$$

In this formula $F(\lambda, \mu)$ is the "power spectrum" of $\zeta(\xi, \eta)$, and $\Phi(\xi, \eta)$ the surface auto-correlation function; F and Φ are each other's Fourier transforms.

The Fourier transform of $\Psi(\xi, \eta)$, called $\Lambda_M(\lambda, \mu)$, has an important physical meaning: it "is proportional to the intensity of waves proceeding parallel to the line with direction cosines λ, ν. ... In general, Λ_M will consist of both a discrete and a continuous portion. The discrete portion, where Λ_M is singular, represents plane scattered waves of finite amplitude (such as the specularly reflected wave). For such plane waves, the integral of Λ_M in the immediate vicinity of its singularity is equal to the square wave amplitude" (Ref. 48, p. 331).

Fourier transformation of Eq. 16 indeed gives two terms of different character:

$$\Lambda_M(\lambda, u) = \Omega(\alpha, \beta)\ \delta(\lambda - \alpha)\ \delta(\mu - \beta) + \Gamma(\lambda, \mu) \quad \text{(Eq. 17)}$$

where Ω is the specular part:

$$\Omega(\alpha, \beta) = 1 - 4\gamma\sigma^2 \iint_{-\infty}^{+\infty} F(\ell - \alpha, m - \beta)\ (1 - \ell^2 - m^2)^{\frac{1}{2}}\ d\ell dm, \quad \text{(Eq. 18)}$$

and Γ the non-specular part:

$$\Gamma(\lambda, u) = 4\gamma^2\sigma^2\ F(\lambda - \alpha, \mu - \beta). \quad \text{(Eq. 19)}$$

This last expression is comparable with Eckart's formula for σ (see Eq. 47). It shows a similar dependence on the wave spectrum F, but differs in the proportionality factor.

The specular term Ω is used in Ref. 49 for the prediction of the surface loss per bounce, when a ray travels in an isothermal surface-bounded channel, whereas the non-specular scattering part Γ, also called "spectral reflection" (Ref. 49, p. 338), is considered in the backscattering studies (Refs. 50, 51). A more detailed treatment of these and related subjects can be found in Refs. 11 and 54. Comparison of the Marsh theory with experimental data shows satisfactory agreement (Ref. 51).

3.2.3 The Dispute about the Rayleigh Method

Commenting upon Rayleigh's procedure for obtaining a solution for the wave equation in the presence of a sinusoidal boundary, Uretsky remarked that: "The crucial and unjustified step in this procedure is the assumption that Eq. 7 describes the solution everywhere above the bounding surface" (Ref. 77, p. 401). Referring to a letter by Lippmann (Ref. 44) he made it seem plausible that the assumption breaks down in the "valleys" between the "peaks", because there both upgoing and down-going waves should be expected. For this reason he carefully developed a solution to the problem, based on Green's theorem. (See Section 3.2.4). Comparing his results with those of Rayleigh,

one of his conclusions (based on numerical experimentation) is " that the Rayleigh equations are useful when the undulations of the bounding surface are gentle (small hK)" (Ref. 77, p. 421).

Meecham too (Refs. 57, 58) remarked that the validity of Rayleigh's second assumption is doubtful. He developed a variational method, for the case of a periodic surface (Ref. 57), which improves the Rayleigh method via an error-minimizing procedure and a Fourier transform method for boundaries of arbitrary shape (Ref. 58). This latter method, in which an approximation of the first derivative of the pressure at the boundary is obtained via a receiver at this boundary, is found to be " preferable to previous methods, notably those which can be classified as physical optics (such as Rayleigh's), since the error in the transform method is of second order in the surface slope whereas the error in previous methods is of first order in the same quantity " (Ref. 58, p. 370 - abstract). Applied to a sinusoidal boundary the method produces expressions for the amplitudes A_m.

The question of the validity of Rayleigh's second assumption has been attacked from another side by Heaps. He presented " an investigation of the least possible value of the surface pressure consistent with the assumption that all the reflected radiation is in the form of undamped plane waves " (Ref. 37, p. 815). He arrived at the conclusion, after comparison of his results with experimental data collected by La Casce and Tamarkin from a sinusoidal model surface (Ref. 41), " that if all the reflected energy has the form of undamped plane waves then the surface is necessarily sound absorbing and of pressure significantly different from zero. Thus, in the neighbourhood of a corrugated surface of zero pressure it is necessary to take into account other forms of radiation and such forms play a significant part in satisfying the boundary condition" (Ref. 37, p. 818). As Marsh has generalized the Rayleigh method for random surfaces, he is arguing "In Defense of Rayleigh's Scattering from Corrugated Surfaces" (Ref. 52). His results (for simplicity he takes a sinusoidal surface) have been compared with those of Uretsky (Refs. 76, 77) by Murphy and Lord. They showed "that Rayleigh's formulation is inadequate for the description of the scattered field" (Ref. 62, p. 1598 - abstract).

The experimental results of La Casce and Tamarkin (Ref. 41) are also used by Uretsky to check his theory. Presenting curves for the reflection coefficients of order $m = 0$ and -1 together with data points. he felt "that the general trend of agreement is encouraging" (Ref. 76, p. 1294).

Also Barnard et al. concluded that the " theory forwarded by Uretsky provides a satisfactory prediction of scattered sound field from a pressure-release sinusoidal surface when the amplitude of the sinusoid is comparable to the wavelength of the incident radiation" (Ref. 13, p. 1169).

Finally we remark that Beckmann (Ref. 2), surprisingly enough, does not touch upon the question of the validity of Rayleigh's assumptions.

The results of the abovementioned papers lead us to the conclusion that the Rayleigh method is indeed incorrect in the way the boundary conditions are used, as it causes contradictions. Nevertheless, for smooth surfaces the method produces results that do not disagree more with experimental data then do other, more rigorous, solutions. It is therefore useful to a limited extent. The method developed by Uretsky, on the other hand, is strict in a mathematical sense and therefore superior to the Rayleigh solution. It is expected to have a much wider validity, as the Rayleigh results can be considered as the first step of a series that converges towards the Uretsky results.

3.2.4 Uretsky's Method for a Sinusoidal Surface

Uretsky devoted two publications to his method: a very short outline (Ref. 76), which is no more than an introduction, and a very thorough and detailed treatment (Ref. 77). The latter contains a complete description of the method with the necessary mathematical proofs, as well as valuable comments upon the Rayleigh method and the Kirchhoff approximation. Application can be found in a study by Barnard et al. (Ref. 13), who summarized the Uretsky approach, made numerical predictions, and compared these with experimental results from a pressure release cork surface in a model tank. Satisfactory agreement was obtained.

The method starts in the same way as the Rayleigh method. A plane monochromatic wave with direction cosines $\lambda_0(= \sin\theta)$ and $\mu_0(= \cos\theta)$ is incident on a sinusoidal pressure release surface

$$z = \zeta(x) = h \cos(Kx). \qquad \text{(Eq. 20)}$$

Instead of assuming that the scattered field can be expanded into an infinite set of plane waves (as Rayleigh did), Uretsky proves that this is possible for observation points not too close to the boundary, i.e.:

$$p_1(x, z) = \sum_{m=-\infty}^{\infty} A_m \exp\left[ik(x \sin\theta_m + z \cos\theta_m)\right] \qquad (z \geq h).$$

$$\text{(Eq. 21)}$$

The difference from Rayleigh appears in the next step: the expansion (Eq. 21) is not valid for $z < h$, because there the terms of Eq. 21 fail to be solutions of the wave equation.

The Helmholtz formula (Ref. 1), which expresses the scattered field p_1 as an integral over elementary sources induced on the boundary by the incident wave p_0, is invoked to avoid Rayleigh's second assumption. In terms of Green's functions* the Helmholtz integral can be written as

$$p_1(\vec{r}) = (4\pi)^{-1} \int \left[G_k(\vec{r}|\vec{r}') \nabla p(\vec{r}') - p(\vec{r}') \nabla G_k(\vec{r}|\vec{r}')\right] dS(\vec{r}'), \qquad \text{(Eq. 22)}$$

where p is the total pressure at the boundary. This quantity is zero on a free surface: hence the second term between square

*A Green's function $G_k(\vec{r}|\vec{r}')$ expresses the field at $\vec{r}$ due to a unit point source at $\vec{r}'$ (monochromatic, wave number k).

brackets in Eq. 22 vanishes. As for a two-dimensional case we have

$$G_k(\vec{r}|\vec{r}\,') = i\,\pi\, H_0^{(1)}\,(k|\vec{r} - \vec{r}\,'|) \qquad \text{(Eq. 23)}$$

the expression for the scattered field becomes:

$$p_1(\vec{r}) = \frac{i}{4}\int_{-\infty}^{\infty} dx'\; H_0^{(1)}\,(k|\vec{r} - \vec{r}\,'|)\;\nabla\, p(\vec{r}\,'). \qquad \text{(Eq. 24)}$$

"The crucial step in the present formulation of the problem is to recognize that $\nabla p(\vec{r}\,')$ admits a Fourier series representation" (Ref. 76, p. 1293); the proof is given in Ref. 77. Hence:

$$\nabla p(\vec{r}\,') = k\sum_{j=-\infty}^{\infty} (i)^{-j}\, B_j\, \exp\left[ik(\lambda_0 + j\,K/k)x'\right]$$

$$\equiv k\sum_{j=-\infty}^{\infty} (i)^{-j}\, B_j\, \exp(ik\lambda_j x'). \qquad \text{(Eq. 25)}$$

Putting this in Eq. 24 gives

$$p_1(x,\ z) = \frac{ik}{4}\sum_{j=-\infty}^{\infty} (i)^{-j}\, B_j \int_{-\infty}^{\infty} dx'\; H_0^{(1)}\,(k|\vec{r} - \vec{r}\,'|)\,\exp\,(ik\lambda_j x'). \qquad \text{(Eq. 26)}$$

In order to find the scattered field p_1, the boundary coefficients B_j have to be determined. This can be done via the boundary condition of zero total pressure, which gives

$$-p_1\big(x,\ \zeta(x)\big) = p_0\big(x,\ \zeta(x)\big) \equiv \exp\Big[ik\big(\lambda_0 x - \mu_0 h\,\cos\,(Kx)\big)\Big]. \qquad \text{(Eq. 27)}$$

As in the Rayleigh method, the expansion

$$\exp(ib \cos Kx) = \sum_{n=-\infty}^{\infty} i^n J_n(b) \exp(inKx) \qquad \text{(Eq. 28)}$$

is used. Writing the Hankel function as an integral

$$H_0(k R_0) = i\pi^{-2} \int \frac{d\vec{K}}{K^2 - k^2} \exp(i \vec{K} \cdot \vec{R}_0) \qquad \text{(Eq. 29)}$$

Uretsky obtained from Eqs. 26 and 27

$$\sum_{m=-\infty}^{\infty} i^{-m} J_m(hk\mu_0) \exp(ik\lambda_m x) =$$

$$\frac{i}{2\pi} \sum_j \sum_n \sum_\ell (i)^{\ell-n} B_j \int_{-\infty}^{\infty} dt \frac{J_{n-j}(hkt) J_\ell(hkt)}{(t^2 - \mu^2_\ell)} \exp(ik\lambda_{n+\ell} x) \qquad \text{(Eq. 30)}$$

where $\mu^2_\ell = 1 - \lambda^2_\ell$ and λ_ℓ is defined in Eq. 25.

Defining a matrix element $M_{\ell j}$:

$$M_{\ell j} = -(2\pi)^{-1} \sum_n (-1)^n \int_{-\infty}^{\infty} dt \frac{J_{n-j}(hkt) J_{\ell-n}(hkt)}{(t^2 - \mu^2_n)} \qquad \text{(Eq. 31)}$$

and equating in Eq. 30 the coefficients of $e^{ik\lambda_m x}$, gives an infinite set of algebraic equations for B_j:

$$\sum_{j=-\infty}^{\infty} M_{\ell j} B_j = (-1)^\ell J_\ell(hk\mu_0). \qquad \text{(Eq. 32)}$$

" The major complication of the problem (other than the usual difficulties associated with inverting infinite matrices) is in the calculation of the matrix elements $M_{\ell j}$" (Ref. 76, p. 1294). But the evaluation is possible, although the result is somewhat complicated:

$$M_{\ell j} = \sum_n (-1)^n \left[R^n_{\ell j}(hk;\mu_n) - i(2\mu_n)^{-1} J_{\ell-n}(hk\mu_n) \cdot J_{n-j}(hk\mu_n) \right]$$

$$(\ell+j \text{ even}), \qquad \text{(Eq. 33)}$$

$$M_{\ell j} = 0 \qquad (\ell+j \text{ odd}).$$

The function $R^n_{\ell j}$ may be written in terms of the generalized hypergeometric function ${}_3F_4$:

$$R^n_{\ell j}(b;\mu) = b\left(\frac{2}{\pi}\right)^2 \frac{{}_3F_4\left(1,1,\frac{3}{2};b,-b,\frac{\ell-j}{2},\frac{j-\ell}{2};\ -b^2\mu^2\right)}{[(2n-\ell-j)^2 -1]\ [(\ell-j)^2 -1]} . \qquad \text{(Eq. 34)}$$

Inversion of Eq. 32 yields the boundary coefficients B_j. A relation between A_m and B_j is then needed for the calculation of the scattered wave p_1 with Eq. 21. The required relation is proved to be

$$A_m = (-i)^{m+1} (2\mu_m)^{-1} \sum_j B_j J_{m-j} (hk\mu_m). \qquad \text{(Eq. 35)}$$

Obviously the Uretsky method is far from simple. But the results are obtained with a high degree of mathematical strictness and with a minimum of conditions on the validity. In fact the only condition is that shadowing does not occur, i.e.

$$\tan\theta \le hK. \qquad \text{(Eq. 36)}$$

It may be noted that only the surface height h appears explicitly in the formulae; the surface period K is still present, however, as it should be via μ_m and λ_m.

A generalization of the Uretsky method for random surfaces seems possible, Marsh having indicated the way to do it. The result could be interesting (although probably rather complicated), as it would be applicable to the ocean surface without too restrictive conditions for the roughness.

3.2.5 Comparison of Several Methods with Each Other and with Experimental Results

It has been noted already that La Casce and Tamarkin have provided the theoreticians with experimental data that could serve as a check for their theories. These data have been published, more than ten years ago, in a study on the reflection of underwater sound from a corrugated surface (Ref. 41). They will be discussed later.

In addition to their experimental work, the authors have summarized the theories of Rayleigh, Eckart (Ref. 26) and Brekhovskikh and compared them with their data. Their formulae for the amplitude coefficients are reproduced in Table 2.

Several authors have used the experimental results of Ref. 41 to check their own theories: Meecham (Ref. 58) applied his Fourier-transform method to a sinusoidal boundary, Parker (Refs. 63, 64) extended the Rayleigh series of plane waves into an integral, Heaps derived from the Rayleigh method a recurrence relation for A_m (Ref. 36) and (with the assumption that the reflected field contains only undamped plane waves) obtained values for A_m that minimize the mean square pressure at the boundary (Ref. 37), and, lastly, Uretsky (Ref. 76) avoided the mathematical defect inherent in Rayleigh's procedure with a careful and rigorous solution.

All the methods developed for the scattering from sinusoidal boundaries, though very different in their final results, agree

TABLE 2

AMPLITUDE COEFFICIENTS FOR A SINUSOIDAL BOUNDARY (Absolute Values)

(From: La Casce and Tamarkin - Ref. 41)

Rayleigh

$$A_0 = J_0(2\ hkc) + \tfrac{1}{2}(C - C_{-1})\ hk\, J_1\ (2hkc)$$

$$A_{-1} = J_1\,(2\ hkc)$$

Eckart

$$A_0 = J_0(2\ hkc)$$

$$A_m = \frac{c + c_m}{2c}\ J_m\left[c + c_m)\ hk\right]$$

Brekhovshikh

$$A_0 = J_0(2\ hkc)$$

$$A_m = \frac{(c + c_m)^2 + (s - s_m)^2}{2\ c_m(c + c_m)}\ J_m\left[(c + c_m)\ hk\right]$$

$(c = \cos\theta,\ s = \sin\theta,\ c_m = \cos\theta_m,\ s_m = \sin\theta_m)$

in predicting that the main directions of scattering are given by the grating formula:

$$\sin \theta_m = \sin \theta + m K/k \,, \qquad \text{(Eq. 37)}$$

where K is the surface wave number and the other quantities are as defined in Fig. 3

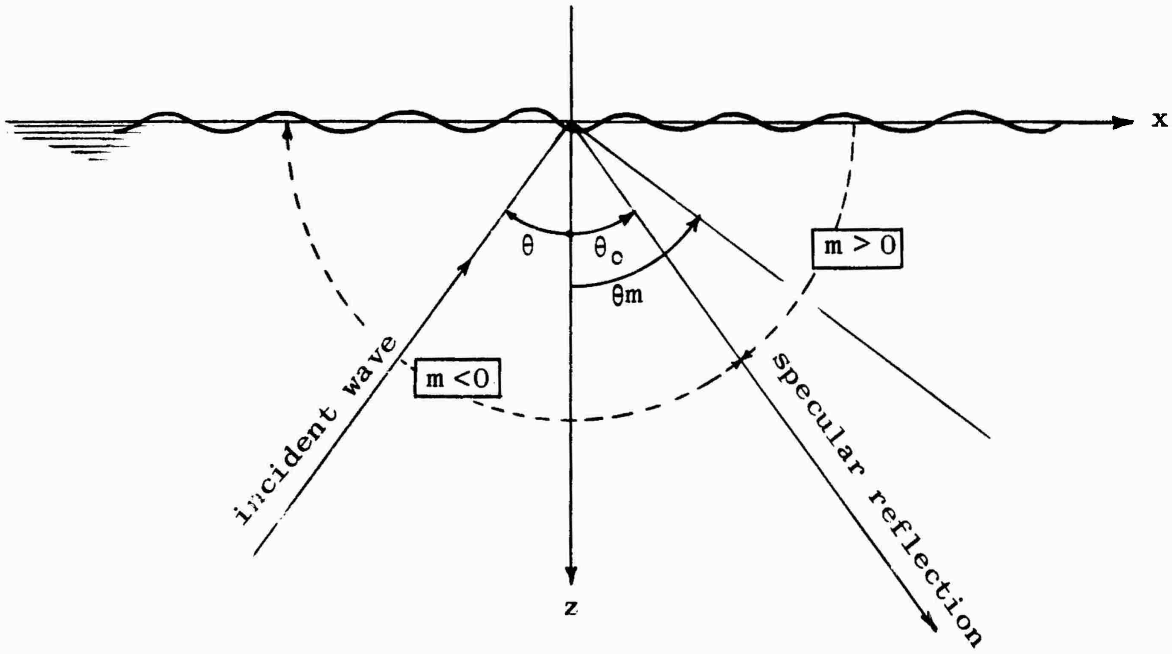

FIG. 3 DEFINITION OF ANGLES AND MODES FOR THE SCATTERING OF PLANE WAVES FROM A SINUSOIDAL BOUNDARY. THE SPECULAR DIRECTION OCCURS FOR m = 0.

TABLE 3

SURFACE PARAMETERS (La Casce et al. - Ref. 41)

Surface	K (cm^{-1})	h (cm)	Kh
1	6.64	0.32	2.12
2	3.12	0.24	0.75
3	3.08	0.15	0.46

La Casce and Tamarkin obtained their results with pressure release cork surfaces of approximately sinusoidal form, floating on top of the water in a tank. Such a surface can be described with the formula

$$\zeta(x) = h \cos (Kx). \qquad \text{(Eq. 38)}$$

For a concrete situation values have to be assigned to the following parameters: θ, k, h, K and m, the scattering mode number (0, ± 1, ...). La Casce and Tamarkin have experimented with three surfaces, for which h and K are given in Table 3. They measured the scattered amplitude A_m (m = 0, -1 and -2) for $\theta = 0°, 20°, 40°$ and 60° as a function of kh, thus providing a rich source of data for comparison.

In order to facilitate the comparison of the available theories with each other and with experimental results, we have plotted in Figs. 4-9 some of the data of La Casce and Tamarkin together with theoretical curves. The ones according to Rayleigh, Eckart and Brekhovskikh we have computed with the formulae of Table 2; the other curves are copied from the original papers.

The figures show the specularly reflected amplitude and the first and second order backscattered amplitudes for $\theta = 0°$ and 40°, as functions of kh, for the third experimental surface (hK = 0.46), as this is the most sinusoidal one and because most of the theories presented are based on the assumption of small surface slopes.

Since the surface under study is not very rough, the Rayleigh prediction is not significantly worse than other curves. The Uretsky curves, for which a small slope is not required, are satisfactory but do not appear superior to the others. More interesting, therefore, is the application of Uretsky's theory to rough surfaces.

This has been done by Barnard et al. (Ref. 13) in their model studies. Their surface can be described with: h = 1.5 cm, $K = 1.4\ cm^{-1}$ and hence hK = 2.1. The frequency of incident sound

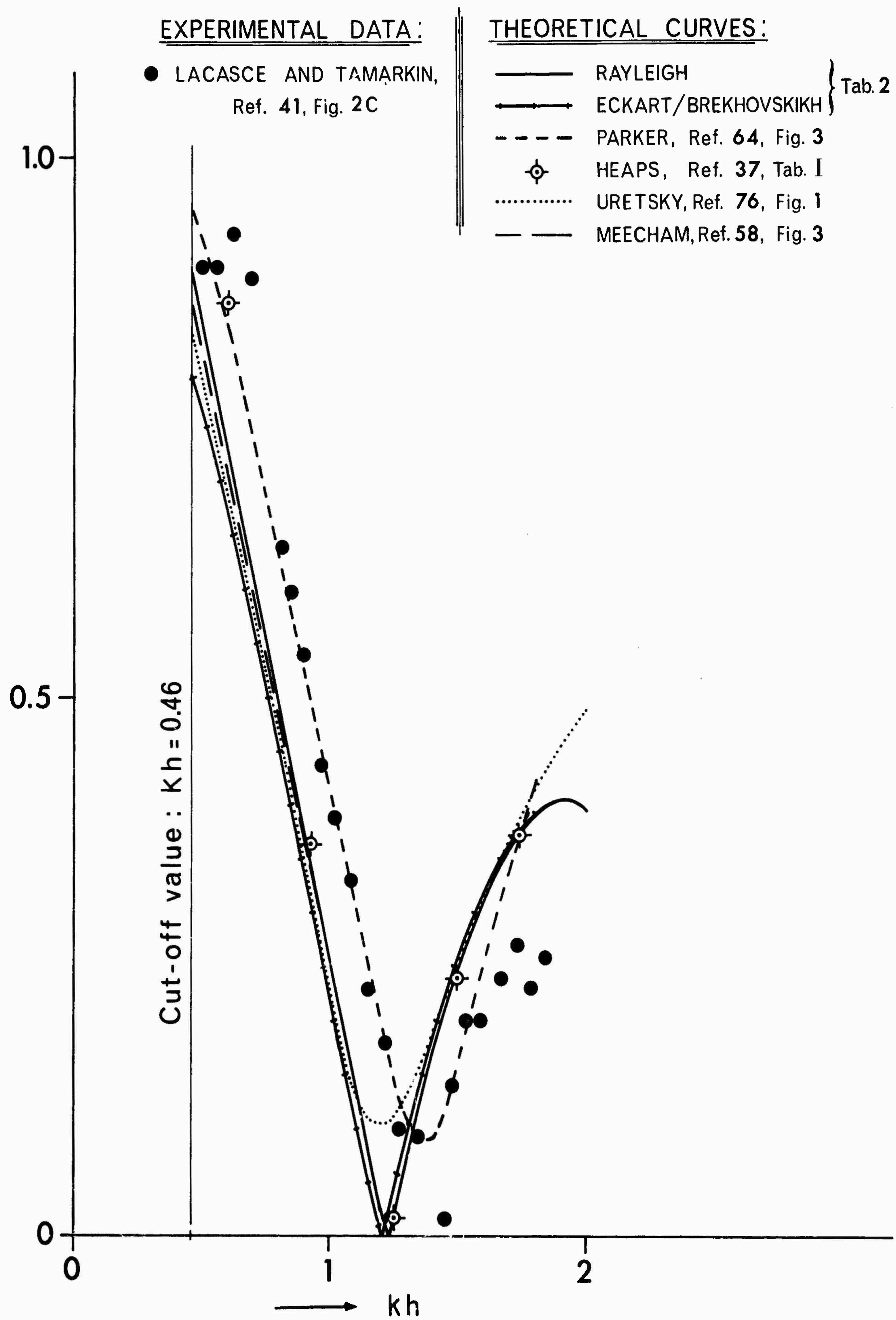

FIG. 4 THE SPECULARLY REFLECTED AMPLITUDE $|A_0|$
Surface roughness $hK = 0.46$, normal incidence $\theta = 0°$.

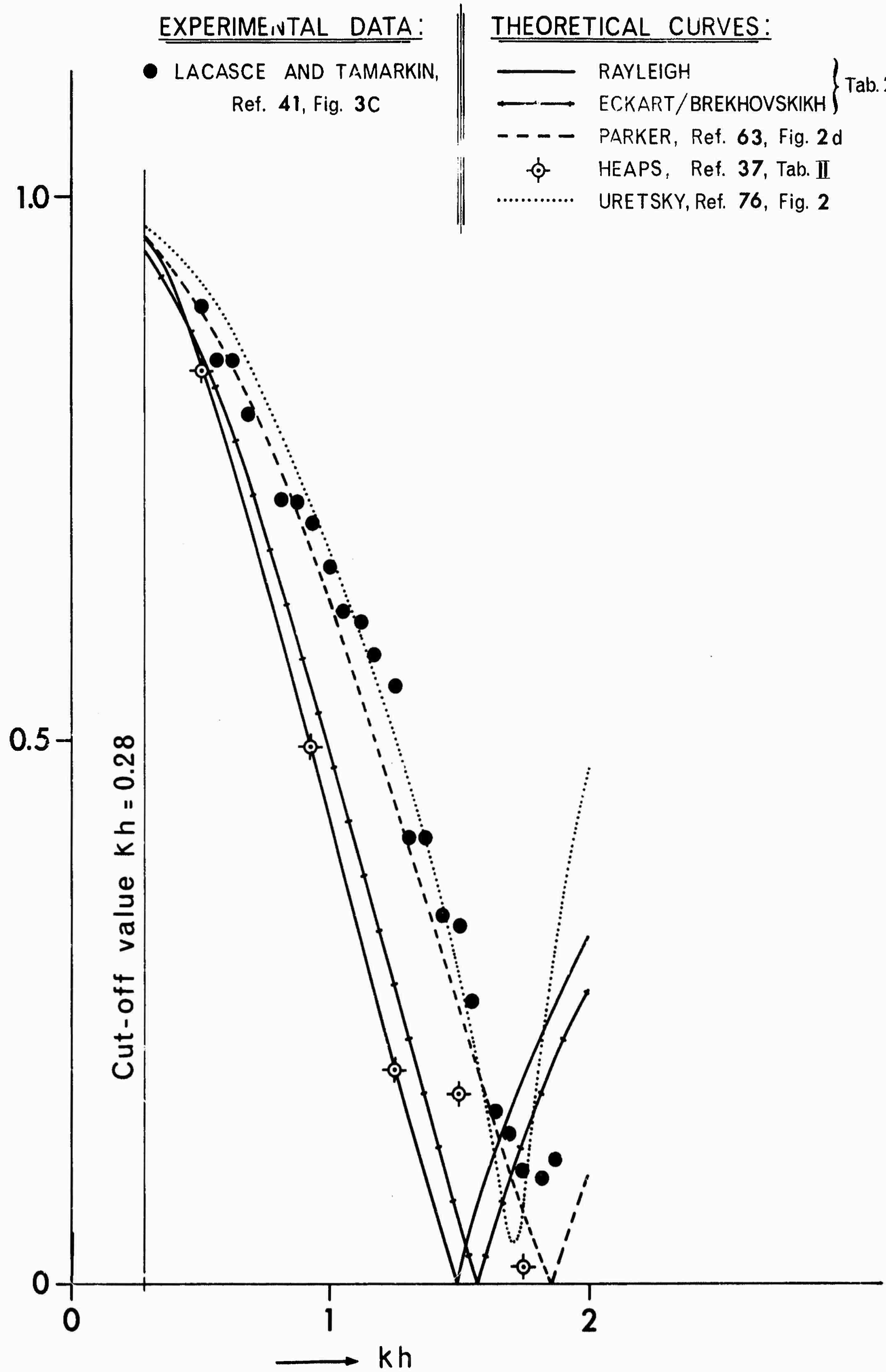

FIG. 5 THE SPECULARLY REFLECTED AMPLITUDE $|A_0|$
Surface roughness hK = 0.46, angle of incidence ϑ = 40°.

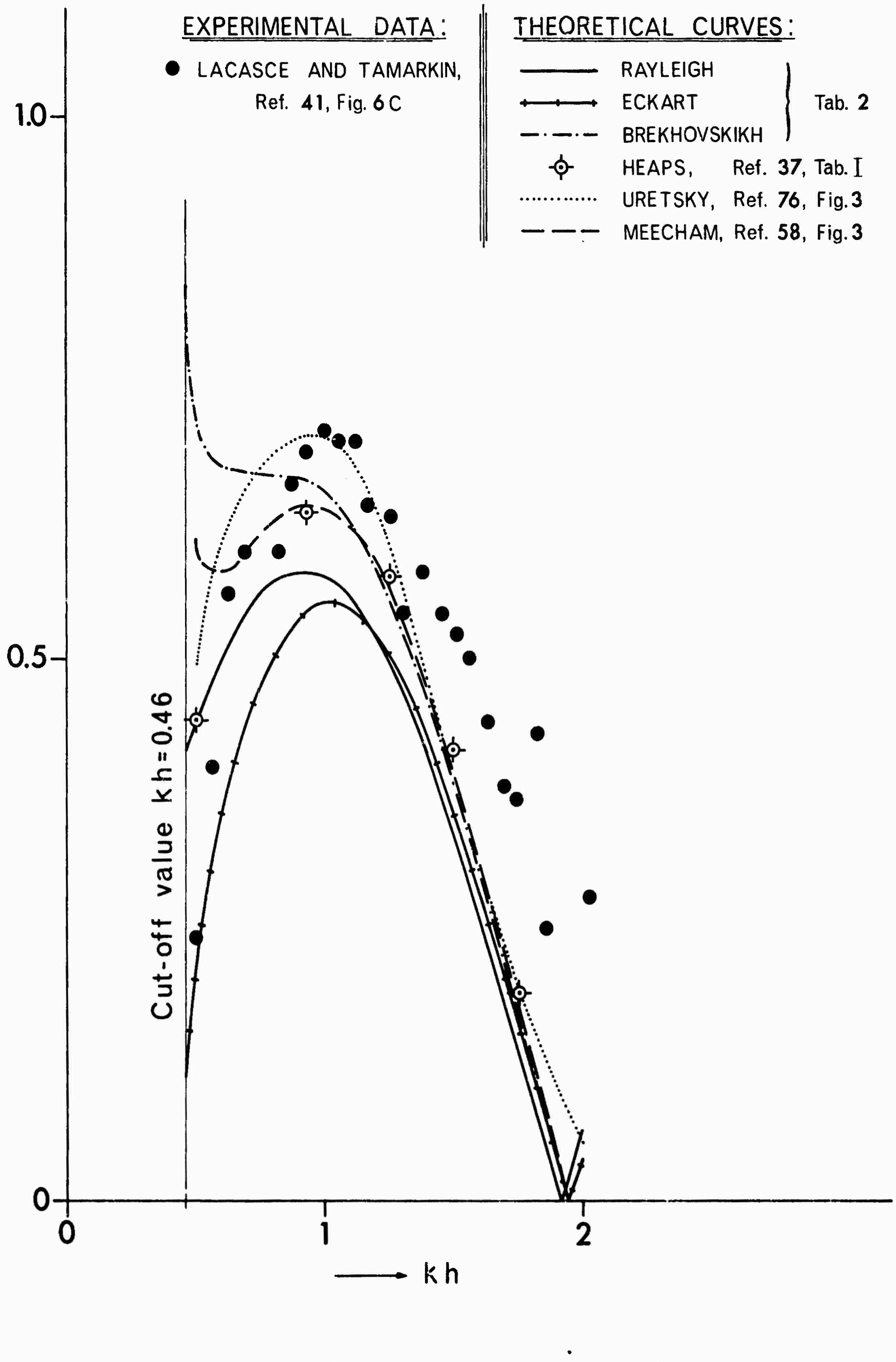

FIG. 6 THE FIRST-ORDER SCATTERED AMPLITUDE $|A_{-1}|$
Surface roughness hK = 0.46, normal incidence $\theta = 0°$.

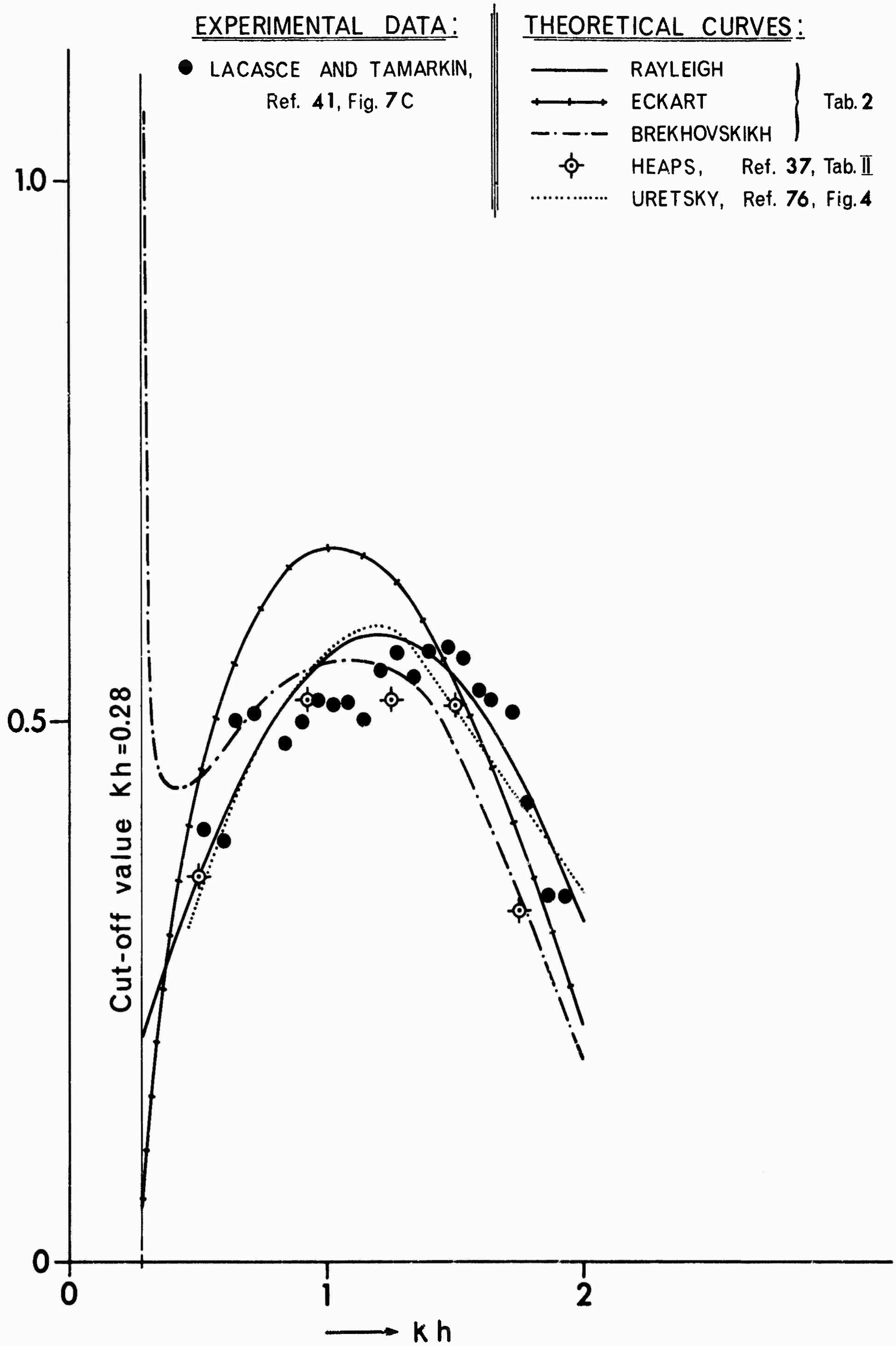

FIG. 7 THE FIRST-ORDER SCATTERED AMPLITUDE $|A_{-1}|$
Surface roughness hK = 0.46, angle of incidence $\theta = 40°$.

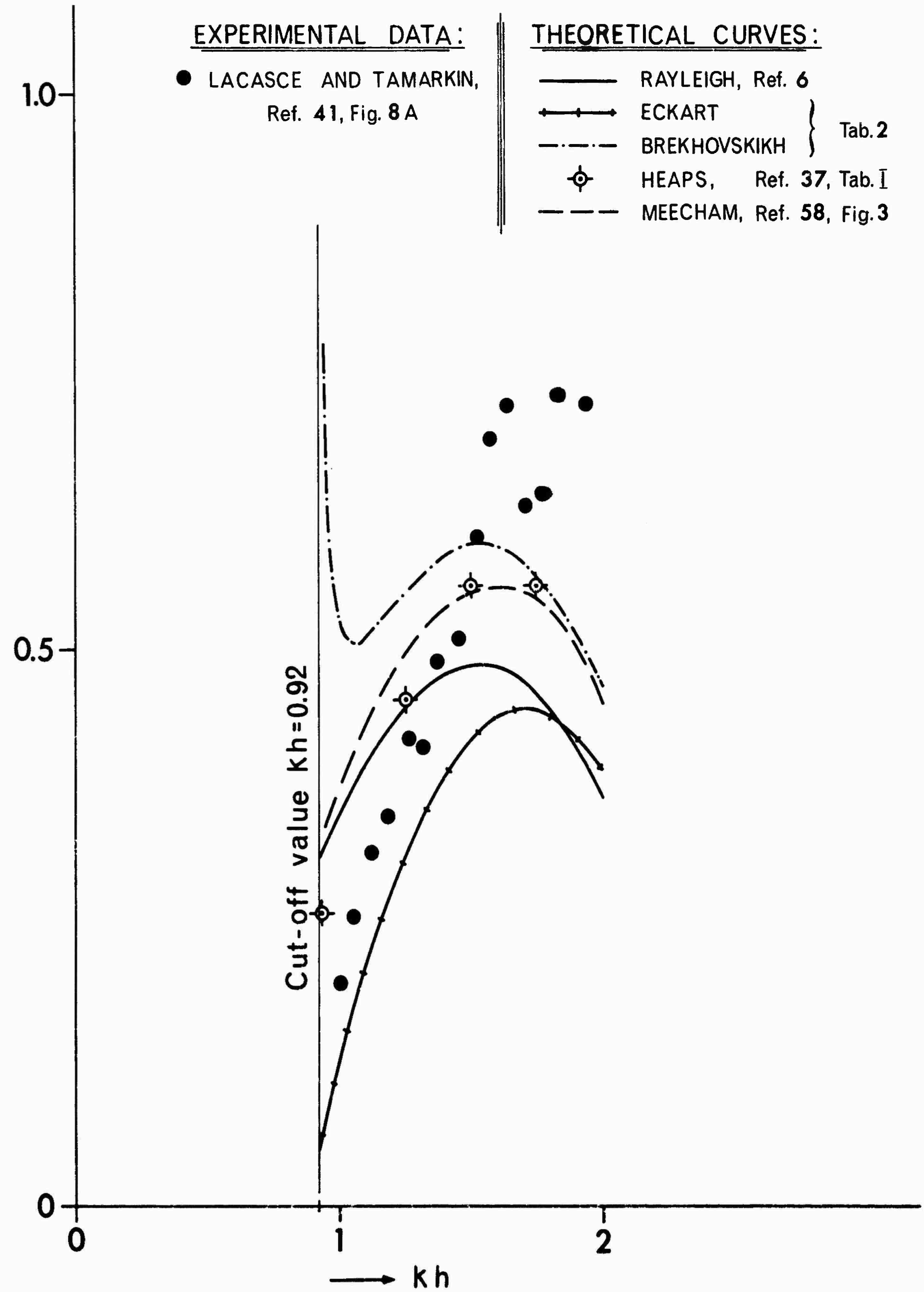

FIG. 8 THE SECOND-ORDER SCATTERED AMPLITUDE $|A_{-2}|$
Surface roughness hK = 0.46, normal incidence $\theta = 0°$.

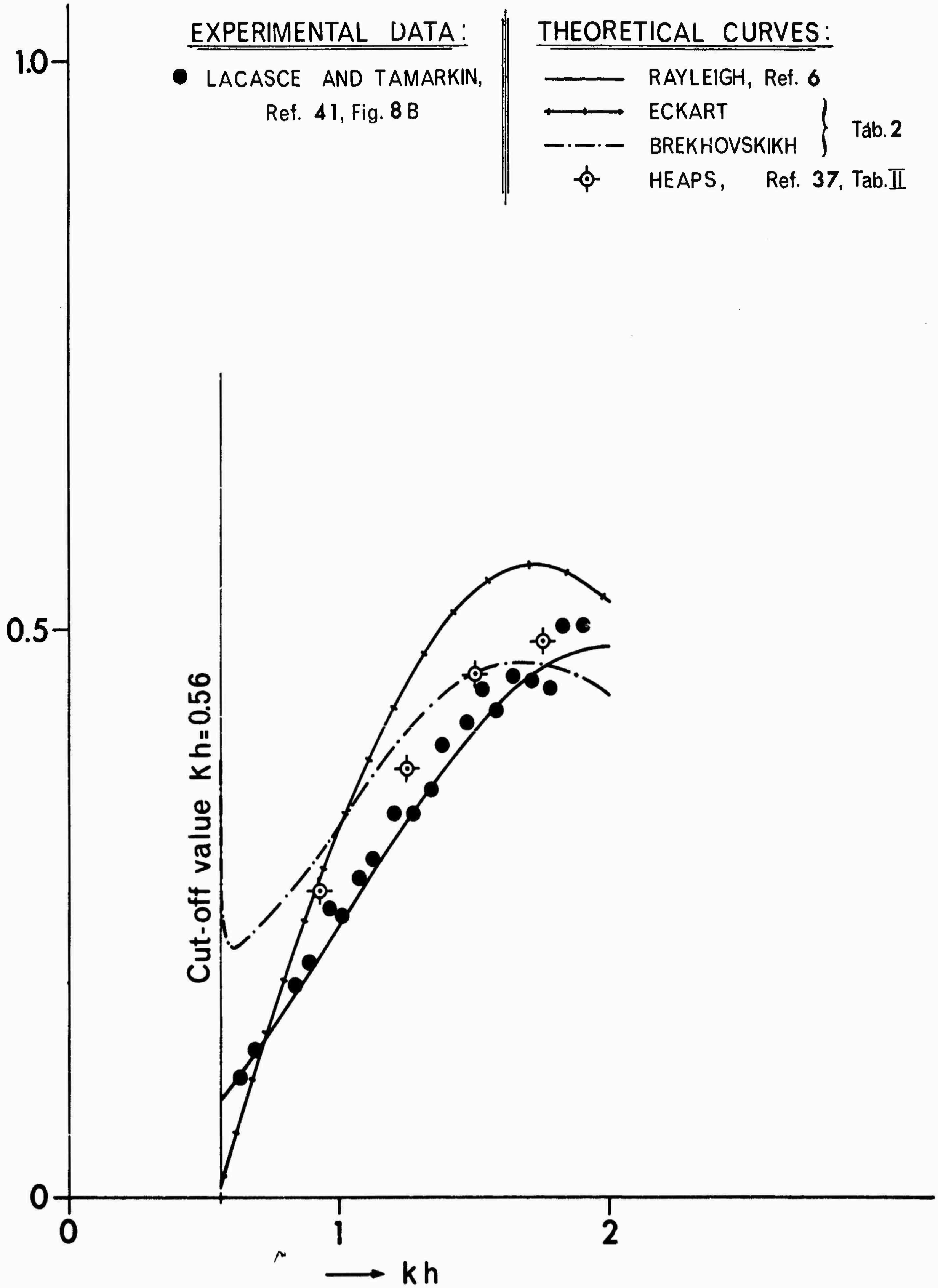

FIG. 9 THE SECOND-ORDER SCATTERED AMPLITUDE $|A_{-2}|$
Surface roughness hK = 0.46, angle of incidence θ = 40°.

was 100 kHz (or $k = 4.2\ \text{cm}^{-1}$, making $hk = 6.3$). They measured the backscattering as a function of grazing angle with fixed angle of incidence. A typical result is shown in Fig. 10. "The agreement between the calculated and experimental curves ... is, in general, excellent" (Ref. 13, p. 1168).

3.3 The Kirchhoff Approximation and Variations

When the scattered field at an observation point is expressed as an integral over elementary sources induced at the surface by the incident wave (this is the so-called Helmholtz integral - Ref. 1), an assumption has to be made for the first derivative of the scattered wave field at the free boundary. For a random surface the exact value of this quantity is hard to obtain; approximation then takes the place of exactness.

The assumption most frequently met is the "Kirchhoff approximation": the required directional derivative is put equal to the first derivative of the incident wave, which is a known quantity.

The leading publication in the group of papers that adopted the Kirchhoff approximation is the paper by Eckart (Ref. 26). The interest of Eckart's work lies in the fact that he "obtained significant results with minimum mathematical complexity by relying on a highly developed physical insight into the problem", as has been remarked by Horton and Muir (Ref. 38, p. 627).

3.3.1 Eckart's Theory

The basic ideas of Eckart's theory can be summarized as follows. A transmitter T (monochromatic) and a receiver are placed above a reflecting surface $S\left[z = \zeta(x, y)\right]$. The transmitter induces elementary sources at S; the scattered pressure field $p_1(R)$ can be obtained from these sources via the Helmholtz integral:

$$p_1(R) = \frac{1}{4\pi}\iint dS\left[\frac{\partial p_1}{\partial n}\ \frac{e^{ikr}}{r} - p_1\frac{\partial}{\partial n}\left(\frac{e^{ikr}}{r}\right)\right] \qquad \text{(Eq. 39)}$$

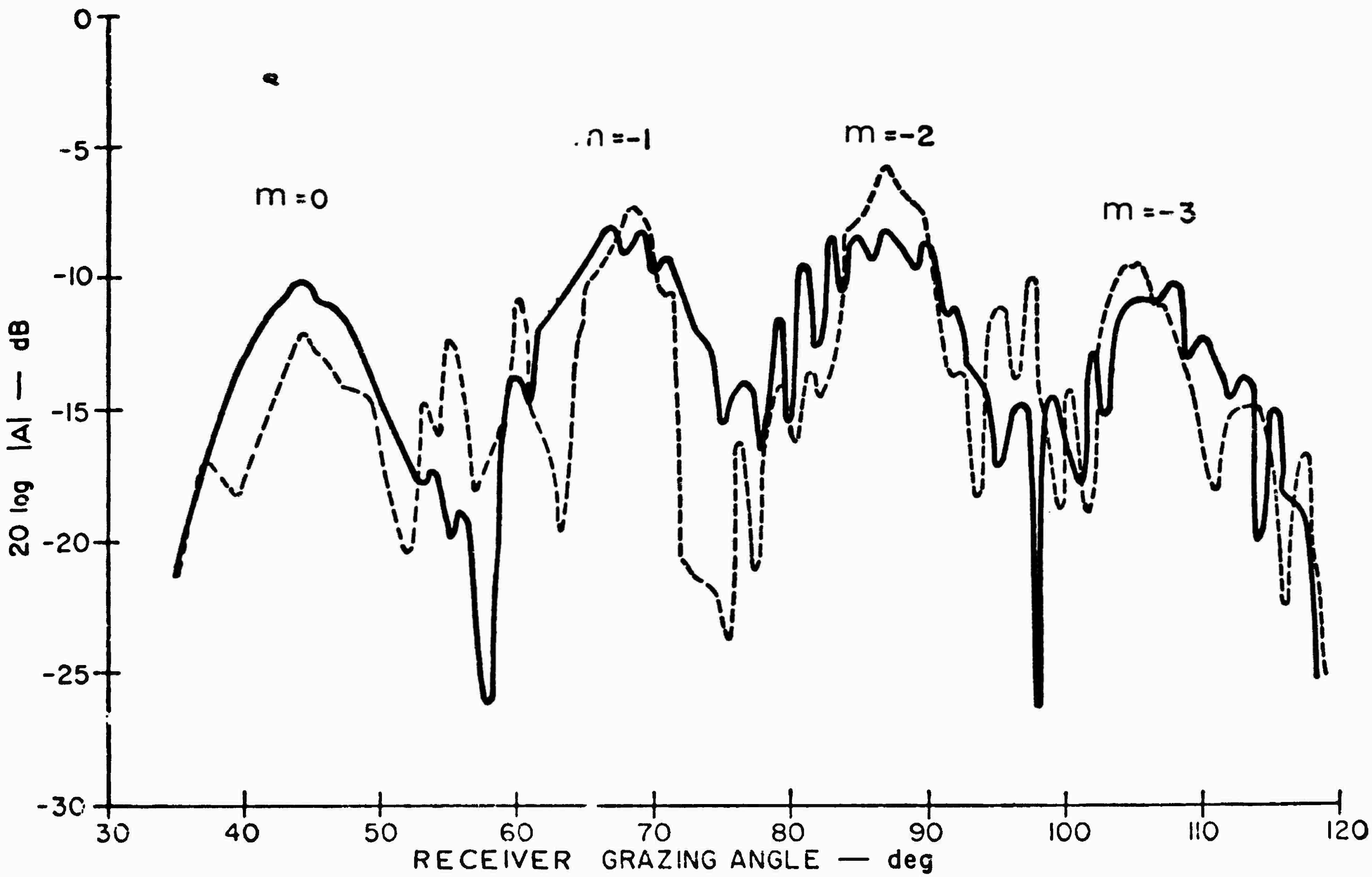

FIG. 10 SCATTERING OF SOUND BY A PRESSURE RELEASE SINUSOID
The scatter order is m. Incident grazing angle, 45°; surface amplitude, 1.5 cm; surface wavelength, 4.5 cm; frequency of incident sound, 100 kc/sec. (From. Barnard, Horton, Miller and Spitznogle – Ref. 13)

in which r is the distance from dS to R, and $\vec{n}$ is the unit normal to dS directed away from R. For a pressure release surface one has the boundary condition

$$p_0 + p_1 = 0 \quad \text{on } S, \tag{Eq. 40}$$

where p_0 is the incident pressure wave. The evaluation of the Helmholtz integral requires also the first directional derivative of p_1. As a second boundary condition Eckart assumed the validity of the Kirchhoff approximation, i.e.:

$$\frac{\partial p_1}{\partial n} = \frac{\partial p_0}{\partial n} \quad \text{on } S. \tag{Eq. 41}$$

Mintzer (Ref. 61) has critizised this assumption with good reasons: when p_1 is fixed on S the quantity $\frac{\partial p_1}{\partial n}$ cannot be chosen independently (Ref. 1). He showed that the second assumption is at most a first approximation for smooth surfaces.

Eckart assumed that T is a directional source and so far away from S that for all points of the insonified area the distance to T is the same. A similar assumption is made for R. Indicating the positions of T and R with the direction cosines $(\alpha_T, \beta_T, \gamma_T)$ and $(\alpha_R, \beta_R, \gamma_R)$, putting $\alpha_T + \alpha_R \equiv \alpha$, etc., and replacing $\frac{\partial}{\partial n}$ by $\frac{\partial}{\partial z}$ (small surface slopes) he derived from Eq. 39:

$$4\pi p_1(R) = ik\gamma \frac{e^{ikr_{10}}}{r_{10}} \iint_{-\infty}^{+\infty} dx\, dy\, P \exp\left[-ik(\alpha x + \beta y + \gamma\zeta)\right] \tag{Eq. 42}$$

where r_{10} is the distance from R to 0, the centre of the insonified area, and P equals the incident pressure at 0. Equation 42 is the basic expression in Eckart's theory. It is used as the starting point for special cases.

Although the Eckart theory can be used for non-random surface profiles (see Section 3.2.5), it is designed originally for a random surface $\zeta(x, y)$ that can be considered as a stationary two-dimensional process, in which case second order moments of the scattered field are calculated.

Two auxiliary functions play a role in the theory:

$$\Phi(\xi, \eta) = \langle \zeta(x, y) \ \zeta(x + \xi, y + \eta) \rangle \qquad \text{(Eq. 43)}$$

and

$$J(\xi, \eta) = \iint_{-\infty}^{+\infty} dx \ dy \ P(x, y) \quad P^{*}(x + \xi, y + \eta). \qquad \text{(Eq. 44)}$$

The function Φ is the autocovariance function of the surface relief and J can be considered as the autocovariance function of the surface insonification.

Putting $\Phi(0, 0) \equiv h^2$, calling a the effective correlation distance of $\zeta(x, y)$ and L the effective size of the insonified area, the basic conditions of Eckart's theory are

$$h \ll \lambda \ll a \ll L. \qquad \text{(Eq. 45)}$$

Eckart calculated the average scattered intensity $\langle I_s \rangle$, for the low frequency and the high frequency cases. For the low frequency case he found

$$\langle I_s \rangle = J(0, 0) \ \sigma \qquad \text{(Eq. 46)}$$

with

$$\sigma = (k^2 \gamma^2 / 4\pi)^2 \quad F(k\alpha, k\beta), \qquad \text{(Eq. 47)}$$

the function $F(K_x, K_y)$ being the surface wave spectrum. He refers to σ as " a dimensionless quantity that may be called the scattering

coefficient, or more descriptively, the scattering cross section for unit solid angle per unit area of sea surface" (Ref. 26, p. 568). Equation 47 indicates an important result: " the low-frequency scattering is determined by the surface spectrum, and not by the height distribution " (Ref. 50, p. 197).

In the high frequency case the calculation of σ is possible only if the characteristic function of the two-dimensional random variable $W \equiv [\zeta(x, y), \zeta(x', y')]$ is known. The hypothesis of a Gaussian probability density yields an expression for σ that is independent of frequency. This is a disappointing result for the "inverse problem" (see Section 4.7) as it does not contain the function Φ but only the variances of the surface slopes.

3.3.2 Variations of Eckart's Theory

Horton and Muir (Ref. 38) extended the low frequency case by specifying $\Phi(\xi, \eta)$ $\left(\text{or } F(K_x, K_y),\right.$ its Fourier transform$\left.\right)$ for isotropic cases. Among others they substituted an exponential and a Gaussian shape for Φ. They found in all considered cases that, if $a \gg h$, "the scattered energy is highly directional and is concentrated about the direction of specular reflection" (Ref. 38, p. 632).

A companion paper by Horton, Mitchell and Barnard (Ref. 39) deals with experiments on a rough Gaussian surface in a model tank. The authors used the high frequency formula for σ of Ref. 38 to check their experimental data. The agreement was not very satisfactory, until they changed the second boundary condition into:

$$\frac{\partial p_1}{\partial n} = 0 \quad \text{on } S \qquad \text{(Eq. 48)}$$

being a compromise between Eq. 41 (valid for illuminated areas) and $\frac{\partial p_1}{\partial n} = -\frac{\partial p_0}{\partial n}$ (holding in the deep shadows). The remarkable effect of this new boundary condition can be observed in Fig. 11.

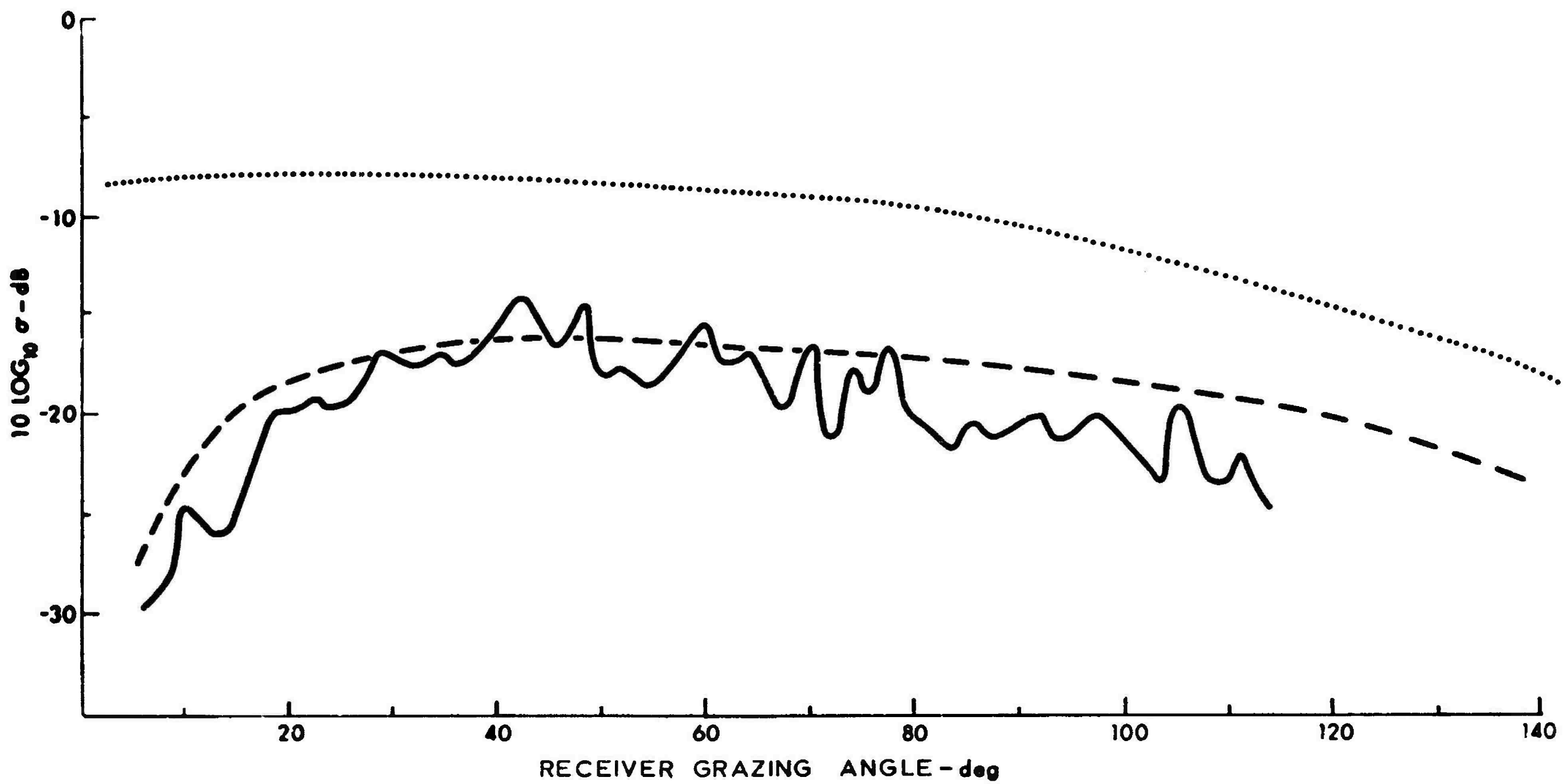

FIG. 11 SCATTERING FROM A ROUGH SURFACE WITH GAUSSIAN CORRELATION FUNCTION

—— Experimental

········ Eckart theory with second boundary conditions given by Equation 41

- - - - Eckart theory with second boundary conditions given by Equation 48

f = 100 kHz, h = 9 mm, θ = 45°. (From: Horton, Mitchell and Barnard – Ref. 39)

Although Eckart discusses only a low frequency and a high frequency case, his theory is also valid in the intermediate range of frequencies. Proud, Beyer and Tamarkin presented "a solution valid for all wavelengths" (Ref. 69, p. 544) for a surface with Gaussian probability density (at least up to the second order), in which the Fourier integral plays an important part.

There is a difference between their procedure and the one followed by Eckart, which may be important for practical purposes at low frequencies. "In the original Eckart theory, the scattering was described in terms of a scattered intensity proportional to the square of the magnitude of the difference in pressure reflected from the rough surface and that reflected from a plane surface replacing the rough one. This procedure dictates that one knows both the amplitude and phase of these pressures in an experimental determination of the scattered intensity". The procedure adopted by Proud et al. "leads to the experimentally simpler operation of forming the difference between plane and rough surface reflected intensity. No consideration of phase is then necessary" (Ref. 69, p. 546).

The authors investigated the dependence of the specular reflected intensity on the acoustic wave number, angle of incidence, and surface roughness. The experimental part of their investigations took place in a model tank with surfaces that had approximately Gaussian characteristics. The quantity $kh\gamma$ ranged from 0.25 to 2.00 in the first case, i.e. from a smooth to a rough surface. The agreement between theory and experiment was good, notwithstanding the violation of the condition of small surface slope.

A comparison between theory and experiments at sea has been made by Clay (Ref. 22). Using the data of Brown and Ricard (Ref. 17), — who placed a pulsed-CW source (168 Hz, 89 ms long) and a receiver at a depth of 1000 yards, varied their horizontal distance between 1000 and 5500 yards, and measured the fluctuations of the scattered field — he found from numerical calculation "a curve that had about the same dependence upon the source-receiver separation as the experimental data" (Ref. 22, p. 1551). Clay extended the Eckart theory to an omnidirectional source by subdivision of the surface in rectangles for which the original theory could be applied.

3.4 The Quasi-Phenomenological Approach of Middleton

In contrast to the most widely employed "physical" methods, where the irregularity of the boundary is introduced via the boundary condition and where the solution of the wave equation has to satisfy this complex boundary condition, the quasi-phenomenological approach of Middleton (Refs. 59, 60) introduces the irregularities of the surface independent of the wave equation as a random distribution of point scatterers, each with its own impulse response function and directivity pattern. This makes the model very flexible from a theoretical point of view: time-variation, frequency dependency of the scattering, broadband signals, complex geometry, and directivity of transmitter and receiver, subsurface scatterers (and also bottom and volume scatterers) are easily incorporated into the model, and there is no limitation on the degree of surface roughness. For this reason Middleton's is the most complete theoretical method. "The critical advantage of this approach are the elimination of impossibly complex boundary conditions, the inclusion of the essential geometry of the overall system, and the ability to handle general signals and aperture distributions. The principal, but not serious, limitation appears to lie in the ultimately empirical nature of the impulse response function of the scatterers, which must be quantified at some stage by experiment" (Ref. 59, p. 374). The problem of how these experiments should be performed is not discussed, unfortunately. For this reason the practical significance of this elegant theory seems limited. The most promising application for our purposes may be found in computer simulations of the scattering phenomenon, via a Monte Carlo method. On the other hand, the physical models, although very limited in their validity, seem to have a closer relation to experimental work.

3.5 Experimental Results

3.5.1 The Amplitude of the Scattered Waves

When a monochromatic sound wave (that is, of constant amplitude) is scattered from a wind driven surface, the amplitude of a diffracted wave shows fluctuations in time due to the time-variation of the reflecting boundary.

This effect has been measured by Liebermann (Ref. 42) and Pollak (Ref. 67) at sea, and by D'Antonio and Hill (Ref. 25) with a model tank.

Liebermann (Ref. 42) swept the frequency of his source from 27 to 33 kHz in 20 milliseconds and observed the interference pattern between reflected and direct wave. He defined a reflection coefficient V as

$$V = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \qquad \text{(Eq. 49)}$$

where A_{max} and A_{min} are the first maximum and the first minimum of the signal envelope, and found that:

a) Surface reflectivity is highly frequency dependent.

b) The median value of V is near to unity, but for approximately 10% of the time $V > 1$ because of focusing by the surface (p. 498 - abstract).

c) No correlation exists between surface wave height and reflection coefficient (p. 503).

Pollak (Ref. 67) used a pulsed-CW source of 100 kHz and analyzed the reflected amplitude statistically. His results indicate that the probability density function of the reflected amplitude follows approximately a Rayleigh curve (Fig. 12). The same result has been obtained by D'Antonio and Hill (Ref. 25) with a wind driven surface in a model tank. They conclude that "(a) for CW transmission, the envelope of the received signal has a bandwidth greater than the bandwidth of the surface amplitude; (b) crosscorrelations observed between envelopes of amplitude-modulated transmission signals and envelopes of the received signals are low but finite; and (c) there is no correlation between the surface amplitude and the envelope of the received signal" (p. 701 - abstract).

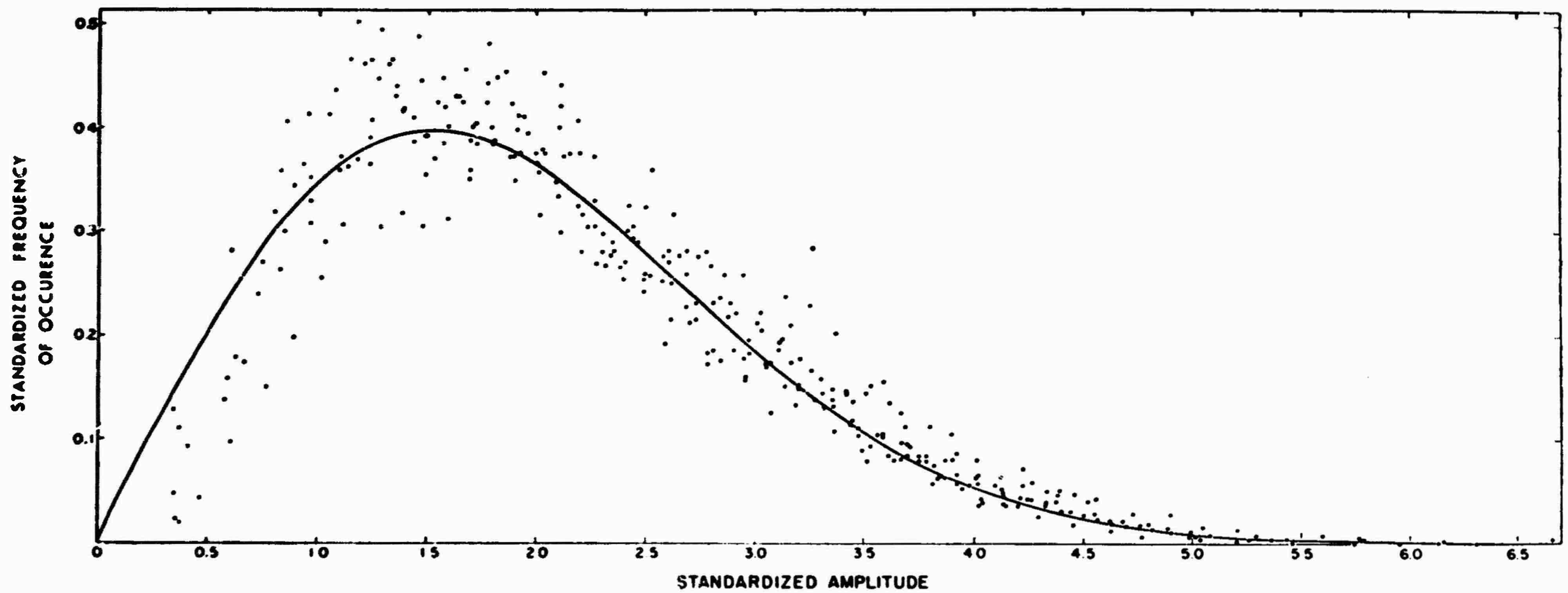

FIG. 12 SCATTER DIAGRAM OF FREQUENCY OF OCCURRENCE VERSUS AMPLITUDE OF WAVES SCATTERED FROM THE OCEAN SURFACE. Standardized coordinates. Points show experimental data. The Rayleigh distribution is represented by the solid curve. (From: Pollak – Ref. 67)

3.5.2 The Intensity of the Backscattered Waves (Reverberation)

"The importance of surface reverberation in the applications of underwater acoustics can hardly be over-emphasized. As a result, measurements of the backscattering of sound from the region of the surface have occupied the attention of numerous observers. These efforts have been of considerable value in attempting to formulate a description of the phenomenon which is adequate for the designer of underwater sonic devices and to reach an understanding of the fundamental mechanisms of scattering at the air-water boundary defining the surface" (Ref. 28, p. 104).

A completely flat surface would only produce reflection in the specular direction. In the case of a rough boundary this specular direction (even if it has only theoretical meaning) separates the backscattering from the forward scattering. This separation has been treated more in detail for the periodic boundary (see Section 3.2.1 and Fig. 3).

In experiments at sea the scattered pressure or intensity is often recorded. For comparative purposes a logarithmic quantity seems more convenient. Hence in most papers a definition of surface backscattering strength (in dB) appeared. And although these definitions differ from one author to another (sometimes attenuation and spreading loss are included (Ref. 28), sometimes a simpler approach is followed (Refs. 78, 79)), their true differences are small enough to make comparison possible, as is borne out by papers of a comparative character (Refs. 28, 55. 70, 72).

As an example of such a definition we mention the one presented by Urick (Ref. 78) for plane waves, because of its simplicity. He defined the backscattering strength, which we shall call σ_B, as

$$\sigma_B = 10 \log (I_s/I_0) \qquad \text{(Eq. 50)}$$

where I_s/I_0 "is the ratio of the scattered intensity from the unit area, measured at unit distance, to the intensity of the incident sound beam. Following naval practice, these distances are expressed in yards" (Ref. 78, p. 136).

Two types of sound sources are met: the directional transducer — mostly operated with pulsed CW (Refs. 17, 28, 32, 34, 39, 56, 67, 68, 69) — and explosives (Refs. 9, 10, 18, 19, 20, 51, 70). In the latter case the "specular direction" has to be inferred from the geometry; the data processing is then carried out via narrow bandpass filters, making them an aggregate of simultaneous "monochromatic" sources.

All experiments considered here concentrate on the measurement of σ_B as a function of one or more of the parameters φ, v and f. Typical results are shown in Figs. 13 and 14.

The curves for σ_B as a function of φ prompted Urick to "divide the angular range from grazing to normal incidence into three regions, in each of which the dominant scattering process seems to be different" (Ref. 78, p. 140). These regions are indicated in Fig. 13.

In Region I the scattering by subsurface bubbles is predominant, at least when f is of the order of 60 kHz: "bubbles can be important at low grazing angles and high wind speeds, in the 60 kHz region, but definitely not at frequencies of a few kilocycles or below" (Ref. 51, p. 243). This scattering is not very dependent on φ for rough surfaces, indicated by a more or less horizontal curve. Increase of wind speed generates more air bubbles and hence increases σ_B. Clay and Medwin agree with this explanation (Ref. 23, p. 2134), but Chapman and Harris doubt its validity, as they do not observe it at 30 knots. They believe "that the scatterers were in a layer of biological origin"(Ref. 19, p. 1596) because a diurnal variation was observed.

"Turning next to Region III, near normal incidence, the slope of the curves in this region and their behaviour with surface roughness suggests that sound is returned by reflection (rather than scattering), probably by small, flat wave-facets oriented normal to the incident sound beam" (Ref. 78, p. 142). An increase of v now decreases σ_B because at the rougher surface less wave-facets have a slope favourable for reflection.

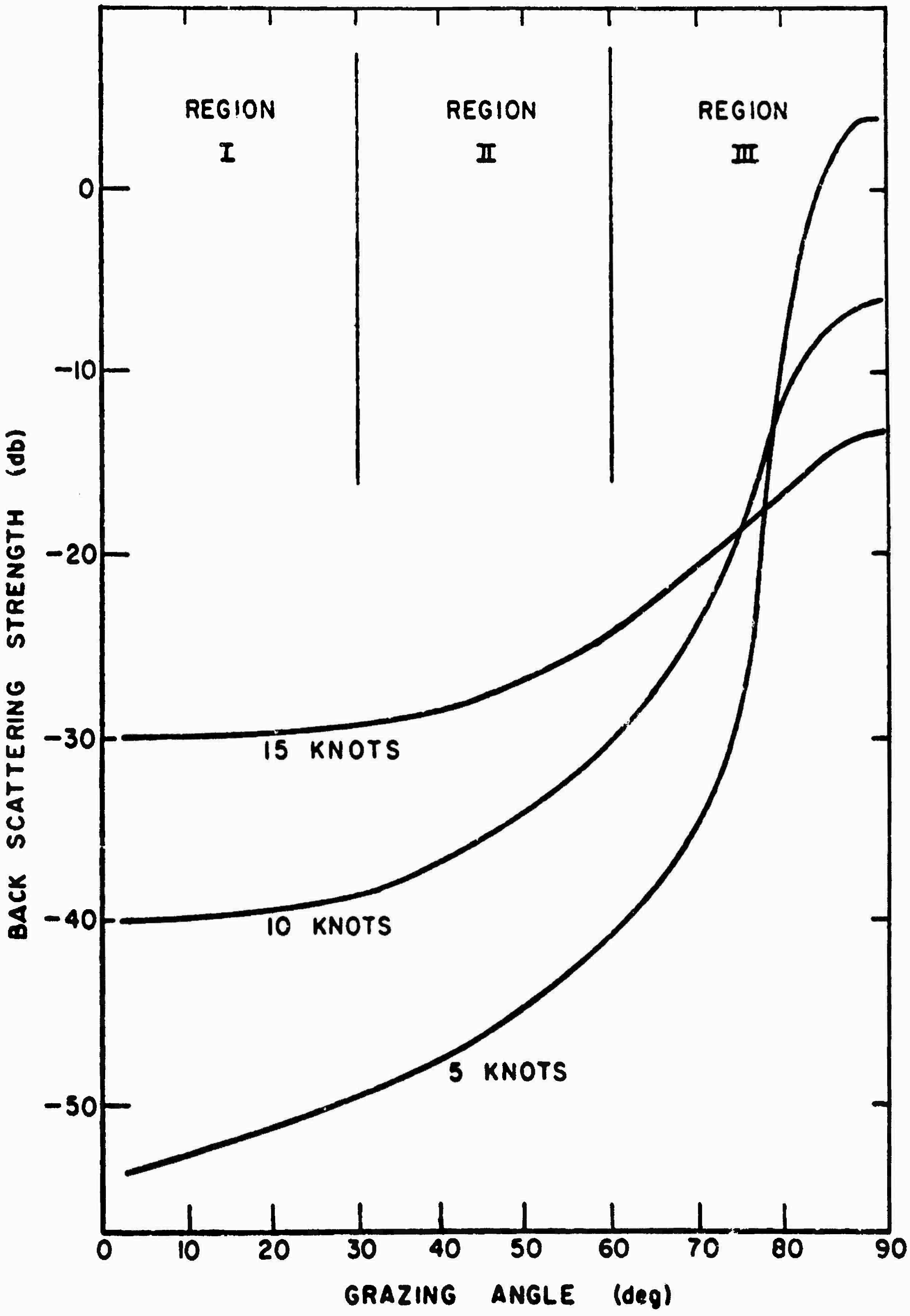

FIG. 13 BACKSCATTERING STRENGTH VERSUS GRAZING ANGLE
Smooth curves of surface scattering strength obtained by averaging field data. Estimated mean deviation of a data point from these curves: 3 db. In the three regions it is suggested that the scattering processes are different.
(From Urick – Ref. 78)

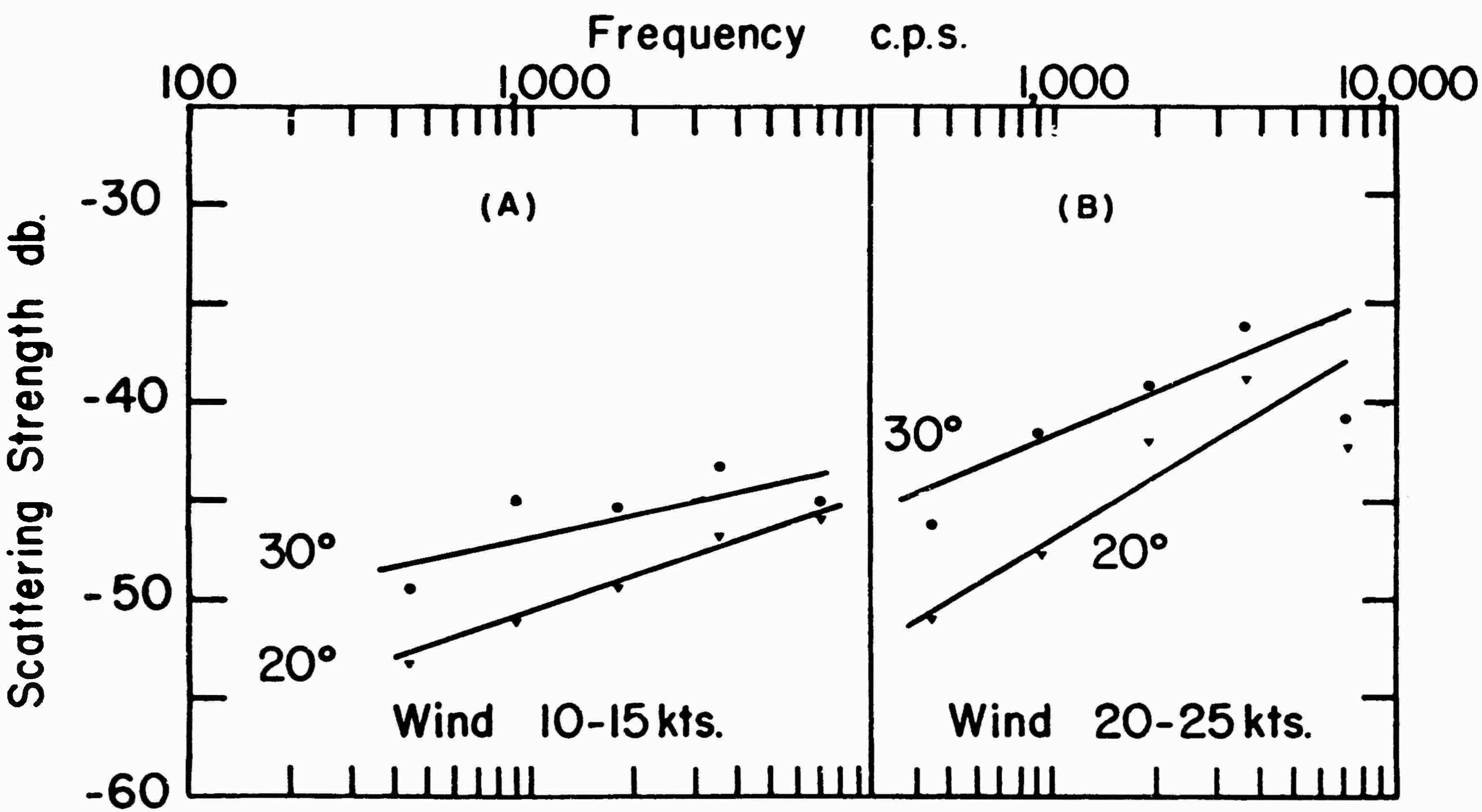

FIG. 14 FREQUENCY DEPENDENCE OF SURFACE BACKSCATTERING
Scattering strength vs frequency at grazing angles of 20° and 30°

(a) Wind speed 20–25 kn
(b) Wind speed 10–15 kn
(From: Brown, Scrimger and Turner – Ref. 18)

In Region II Urick is tempted " to speculate that the slow rise of σ_B with angle in this region represents the effect of roughness scattering by surface irregularities that are much smaller than a wavelength " (Ref. 78, p. 145).

Figure 15 illustrates Urick's theory of regions. "Except for the small angle region", Garrison et al. confirmed this hypothesis (Ref. 28, p. 111). Richter (Ref. 70) reported a σ_B decreasing with φ, and Patterson (Ref. 66) derived a theoretical (phenomenological) model that produces curves similar to those of Fig. 13 in Regions II and III. His curves do not show a constant behaviour in Region I, but this can be explained by the fact that Patterson only dealt with " 'facets' having random distributions of size and slope" (Ref. 66, p. 1150) and neglected bubbles.

In Fig. 14, σ_B is shown as a function of frequency. A proportionality of σ_B with f is indicated. This is in keeping with the results of Chapman et al. (Refs. 19, 20) and Richter (Ref. 70) as is remarked by Brown et al. (Ref. 18, p. 3). On the other hand, in Marsh's theory of backscattering there appears to be an inverse dependence of σ_B on frequency (Ref. 50, Figs. 11-1 and 11-2). Also worth mentioning is that the results of Chapman and Harris are in qualitative agreement with Eckart's theory: at relatively low frequencies σ_B decreases rapidly with decreasing f (see Eq. 47), whereas σ_B is independent of f when f is relatively high (Ref. 19, p. 1594).

An interesting study has been made by Schulkin and Shaffer (Ref. 72). They reviewed experimental results on backscattering in their relation to the Rayleigh criterion of surface roughness ($h \sin \varphi < \lambda/8$). As most of the data are presented as a function of v rather than h, they employed the Neumann-Pierson surface wave spectrum for a fully risen sea in order to relate h and v:

$$2h \equiv H = 0.0026\, v^{5/2} \quad . \qquad \text{(Eq. 51)}$$

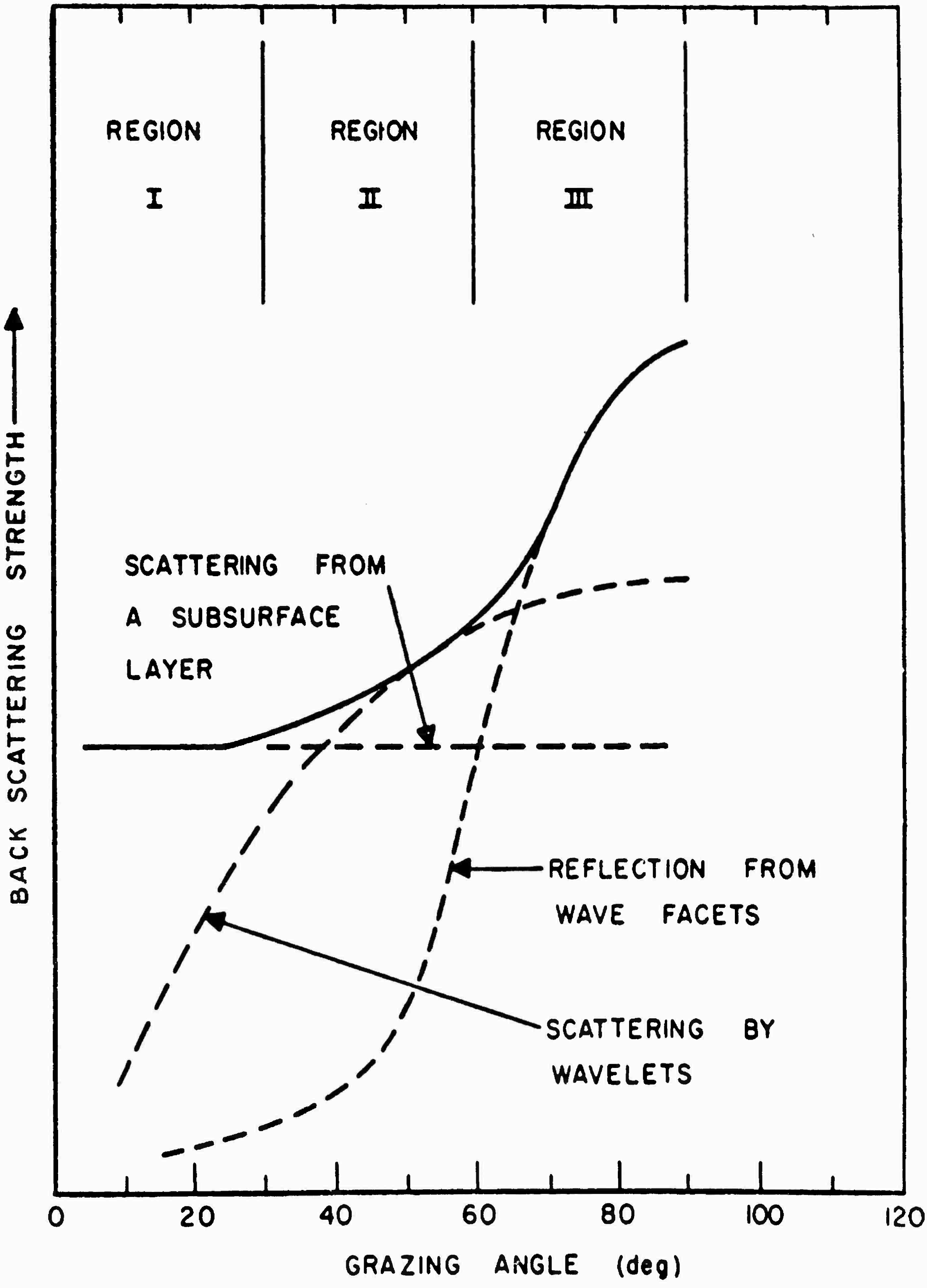

FIG. 15 BACKSCATTERING STRENGTH VERSUS GRAZING ANGLE
Summary of processes advanced as explanations for observed curves of scattering coefficient as a function of angle. (From Urick – Ref. 78)

where H is the mean trough-to-crest wave height (in feet) and v the wind speed (in knots). Then putting

$$\sigma_B = 10 \log(f \quad H \sin \varphi/C)^b, \qquad \text{(Eq. 52)}$$

they calculated the constants C and k for a number of cases [(Refs. 20, 28, 79), plus data from an NDRC report] by drawing the best-fitting straight line through the data. As a result they found that b, the most significant parameter, varied between the values 1 and 2. They concluded that "there is no theory to date to relate all the backscattering-strength data satisfactorily" (Ref. 72, p. 1703).

The differences in the results of backscattering measurements are not only caused by differences in technique or in the definition of σ_B. A factor of great importance, which has not always been recognized by the interpretation of data, is the state of development of the sea surface, which strongly influences the scattering and reflection properties of the sea surface. More details can be found in Section 4.8.

An operational model for sea surface roughness and acoustic reverberation, in which the theory of ocean wave spectra has been applied extensively, has been presented by Martin (Ref. 55). He distinguished scattering and reflection, more or less corresponding to Urick's regions II and III, and combined them into a "total reverberation coefficient". "The model, which has a physical basis over the whole range of incidence angles, is uncertain in its application mainly in present knowledge of the statistics of surface elevation and of derivatives, yet correlates available experimental data about as well as other attempts.

"There is apparently a physical basis for an upper limit of sea surface diffuse-scattering strength (as contrasted to reflection strength) at large wind speeds but in the absence of white-caps and foaming. However, at high enough frequency (kilocycle per second range) and low enough wind speed (5 kn or less), scattering strengths may be 10 dB or more below this limit, especially at small grazing angles. As scattering strengths approach this limiting value

owing to increasing wind speed, there is a rapid (fourth power) increase with wind speed. The frequency effect on scattering strengths is less pronounced than wind-speed effect, there being at most only a moderate decrease (inverse square root) with increasing frequency at low wind speeds. Surface specular-reflection strengths appear to be greatly larger at near-normal incidence than diffuse-scattering strengths; a maximum of reflection strength at moderate wind speed, with limiting values at very low and very high wind speeds, appears clear, but frequency dependence theoretically developed cannot yet be proved by experiment". This quotation from Martin's study (Ref. 55, pp. 706, 707) illustrates the complexity of the phenomenon of sea surface backscattering, and the role of the various parameters.

4. COMMENTS ON SPECIAL SUBJECTS

4.1 Introduction

The subjects discussed in the previous chapter can be considered to be the main currents in the literature.

There are, however, a number of studies that only touch these basic subjects in passing, or that concentrate on a very special aspect. These papers will now be considered.

The last section in this chapter has an oceanographic, rather than an acoustical character, as all the others have, since it deals with the spectrum of the surface waves, and with their height and slopes. But these subjects play an important part in many papers: the height and slopes because they characterize the surface roughness, the wave spectrum because it provides the most realistic way to obtain an expression for the correlation function of the surface irregularity.

4.2 Amplitude and Phase Fluctuations

" The reflection of an acoustic signal from an uneven, time-variant surface leads to variation in the signal form. For a monochromatic wave these variations appear as amplitude and phase fluctuations" (Ref. 21, p. 88).

In previous sections we have seen that for relatively smooth surfaces the total scattered field p_T can be separated into a specularly reflected wave p_r and a diffusely scattered wave p_s. Formulae for p_r and p_s can be obtained (for instance from Eq. 42), by taking the first two terms of the power series expansion of $e^{-ik\gamma\zeta}$. The ratio p_s/p_r is hence a known quantity.

Expressing the pressure p in amplitude and phase $(p = Ae^{i\psi})$ and following Chernov's almost classical work (Ref. 3), amplitude and phase fluctuations can be defined as

$$\frac{\delta A}{A_r} = \mathrm{Re}\left(\frac{p_s}{p_r}\right), \; \delta\psi = \mathrm{Im}\left(\frac{p_s}{p_r}\right) \qquad \text{(Eq. 53)}$$

where it is supposed that $|p_s| \ll |p_r|$. This definition is employed by Gulin and Malyshev (Refs. 30, 31, 32, 33, 34) for the surface diffraction.

An important role in these papers, and also in the work of Smirnov and Tonakanov (Ref. 74), is played by the Rayleigh roughness parameter χ:

$$\chi = 2kh \sin \varphi \qquad \text{(Eq. 54)}$$

(cf Eq. 9).

Gulin started with the Helmholtz integral in the theoretical part of his work (Refs. 30, 31, 33) and used both the Kirchhoff approximation (Refs. 30, 31) and the method of small perturbations (Ref. 33). In the latter case the spatial correlation of the fluctuations was studied. Comparison with experimental data has been made in Refs. 32, 34.

Two different surface correlation functions appeared in the theory:

$$\Phi_1(\xi) = h^2 \exp(-\xi^2/a^2) \cos (K\xi) \qquad \text{(Eq. 55)}$$

and

$$\Phi_2(\xi, \eta) = h^2 \exp[-(\xi^2 + \eta^2)/a^2]; \qquad \text{(Eq. 56)}$$

Φ_1 is an approximation for a quasi-harmonic surface ("swell"), Φ_2 for "sea". Together with these functions the wave parameters D_x and D_y are used:

$$D_x = \frac{ka^2 \sin^2 \varphi}{R_0} \quad , \quad D_y = \frac{ka^2}{R_0} \qquad \text{(Eq. 57)}$$

The transmitter and receiver are lying in the plane $y = 0$, R_0 is half the distance between them via the specular path.

Two regions of D_x are considered: either much smaller than unity, or much larger. The physical significance thereof is that a, the effective surface correlation distance, is either much smaller ($D_x << 1$) than the projection in X-direction of the diameter of the first Fresnel zone along the propagation path (i.e. $\sqrt{2\lambda R_0}/\sin\varphi$), or much larger ($D_x >> 1$).

If L_x, L_y are the dimensions of the insonified surface area, and σ_A, σ_ψ the standard deviations of the fluctuations as defined in Eq. 53, then the results for one receiver can be summarized as follows. Similar conclusions are drawn in Ref. 74.

a) The quasi-harmonic surface Φ_1 gives for $D_x << 1$, a σ_A and σ_ψ that are approximately equal, and proportional with frequency for high frequencies (Ref. 30).

b) The Gaussian surface Φ_2 also predicts for $D_x << 1$ equality of σ_A and σ_ψ (Ref. 30); for low frequencies

$$\sigma_A^2 \simeq \sigma_\psi^2 \simeq \frac{2}{\pi} k^4 h^2 a^2 \sin^4\varphi \, L_x L_y, \qquad \text{(Eq. 58)}$$

showing the same frequency dependence as the Eckart theory (see Eq. 47), but for high values of f there is still a frequency dependency, in contrast to Eckart:

$$\sigma_A \simeq \sigma_\psi \simeq \chi/\sqrt{2}. \qquad \text{(Eq. 59)}$$

This last result, indicating a diffraction grating, has also been found experimentally for $|\chi| \leq 0.7$ (Ref. 32).

For large values of $D_x (>> 1)$ "the amplitude fluctuations are an order of magnitude smaller than the phase fluctuations" (Ref. 30), but still proportional with χ.

c) The probability density function for the amplitude, calculated from experimental data (Ref. 32), ("swell", $\chi > 1$, pulsed CW), confirmed the results of Pollak (Ref. 67) and D'Antonio and Hill (Ref. 25), who obtained approximately a Rayleigh curve. For $\chi < 1$ a Gaussian curve was found to be a good approximation.

d) Time autocorrelation functions, calculated with the same experimental data (Ref. 32), indicated for $\chi < 1$ a decaying periodical behaviour, comparable to Φ_1.

A sinusoidal surface, moving with constant velocity, has also been analyzed (Ref. 31). Rayleigh's theory is used $(m = -1, 0$ and $+1)$. For $\chi \ll 1$, σ_A and σ_ψ have the same period as the surface wave; their magnitudes are equal and proportional to χ. The maxima of σ_A coincide with the minima of σ_ψ, and vice-versa.

As for the spatial autocorrelation, these have been studied theoretically (Ref. 33) and experimentally (Ref. 34). In these papers T and R (i.e. the first receiver) are placed in the plane $y = 0$, at depths z_T and z_R, and at a horizontal distance ℓ that is not smaller than z_T or z_R. The surface is of the quasi-harmonic type, characterized by a correlation function similar to Φ_1, but the waves travel in a direction that makes an angle α with the X-axis. Only correlation distances much smaller than ℓ have been considered and χ was assumed to be less than unity. Three cases have been treated:

a) <u>Correlation in X-direction</u>

For surface waves in X-direction $(\alpha = 0)$ and $D_x \gg 1$ correlation functions $B_{AA}(\rho_x)$ and $B_{\psi\psi}(\rho_x)$ are predicted that have a shape similar to Φ_1. The effective correlation distance a_x, defined with $B(a_x)/B(0) = e^{-1}$, depends on the geometry but is larger than a in most cases. Experiments confirm this (Ref. 34). If $D_x \ll 1$ and $K/k \ll 1$ the normalized autocorrelation functions become

$$B_{AA}(\rho_x) \simeq B_{\psi\psi}(\rho_x) \simeq \tfrac{1}{2}\left[1 + e^{-\rho_x^2/a^2} \cos (K\rho_x)\right] \qquad \text{(Eq. 60)}$$

which assumes the value $\frac{1}{2}$ for $\rho_x \gg a$.

b) Correlation in Y-direction

The case $\alpha = \pi/2$ and $D_x >> 1$ yielded results similar to (a). The interesting case $\alpha = 0$, however, has not been considered.

c) Correlation in Z-direction

With $\alpha = 0$, $D_x << 1$ and $K/k << 1$ the correlation functions become

$$B_{AA}(\rho_z) \simeq B_{\psi\psi}(\rho_z)$$

$$\simeq \tfrac{1}{2}\left[1 + \exp\left(-\frac{\rho_z^2 k}{2a^2 K}\right) \cos\left(\rho_z\frac{\sqrt{hK}}{2}\right)\right] \qquad \text{(Eq. 61)}$$

so that the correlation in Z-direction decreases much faster than in X-direction. This has been observed at small grazing angles for B_{AA} (Ref. 34).

An important conclusion can be drawn from all these cases: both theory and experiment demonstrate the presence of a distinct correlation between the scattered field at one or more receivers and the state (period, roughness) of the diffracting surface. For the reflection coefficient, such a correlation has not been found (see Liebermann's third conclusion, Section 3.5.1).

An extension of Gulin's paper (Ref. 33) to the case of a narrowband signal has been published by Chuprov (Ref. 21). His results are closely related to Gulin's.

4.3 Surfaces with Two Types of Roughness

At the surface of the ocean the roughness can very often be considered as a superposition of several types of roughness: " the typical sea surface is comprised of "swell", "sea" and "ripple" " (Ref. 35. p. 599). In two papers a model with two types of roughness (large-scale plus small-scale) has been developed.

Kur'yanov (Ref. 40) supposed the two types to be uncorrelated. The small-scale irregularities have been assigned a correlation function $\Phi(\rho) = 2\, I_1(\rho/C)/(\rho/C)$, where I_1 is a modified Bessel function. "The choice of this form of correlation function has no particular significance, but it considerably simplifies the subsequent calculations" (Ref. 40, p. 254).

For the coarse surface a sinusoidal and a random one with Gaussian correlation function were taken. In both cases a factor Q was calculated that expressed for a plane wave with grazing angle φ the difference between scattered intensity from the small-scale irregularities on the coarse surface and that from the small irregularities on a flat plane. This factor is about 1 for $\varphi \sim 90°$, but increases rapidly for $\varphi < 60°$.

More realistic is the paper by Hayre and Kaufman (Ref. 35). These authors considered correlated roughnesses, with a normal distribution (four-dimensional), representing a statistically isotropic surface. They calculated the mean scattered power in an arbitrary direction when a plane monochromatic wave was incident. For a slightly rough surface this scattered power contains two terms: a specular and a diffuse one, the latter containing the effect of both types of roughness plus their combined effect, in a rather complicated way. These effects are expressed in second order quantities (variances and correlation coefficients). A moderately rough surface produces additional terms of a more complicated structure. The result of the last case, the "extremely" rough surface (but the Kirchhoff approximation is used and hence the surface cannot be too rough), can be interpreted as if the surfaces consisted of three independent processes: small-scale, large-scale and a combined roughness.

4.4 Surfaces with a Sublayer

Below a wind-driven surface air bubbles are often formed. Moreover, at sea sound speed can vary with depth and biological objects can also be present just below the surface. Consequently the scattering of sound waves from the boundary can be accompanied by a sub-surface scattering.

In particular, Russian authors have tried to find out under what conditions this layer effect can become so important that it "screens" the surface scattering. In most cases this is done via a modified Rayleigh approach. Glotov and Lysanov (Ref. 29) assumed a homogeneous layer of air bubbles whose diameters are small compared with the incident wavelength. Lysanov (Ref. 46) characterized the inhomogeneous layer by the index of refraction $\mu(z)$ and also studied the effect of a layer for which the sound speed is a function of depth (Ref. 47):

$$c(z) = \frac{c_0}{\sqrt{1 - b(z-\Delta)}} \qquad (z \leq \Delta). \qquad \text{(Eq. 62)}$$

In this last case the scattering possesses a resonance character: the reflection coefficient shows peaks " whenever the scattered wave turns out to be a natural vibrational mode for the given layer " (Ref. 47, p. 70).

The results of the work of Glotov and Lysanov (Ref. 29) have some interesting aspects. The authors assume a statistically homogeneous layer of small air bubbles from the boundary down to a depth $z = \Delta$, on which a plane wave is incident with grazing angle φ. The concentration of air bubbles is characterized with a parameter ε. As for the "screening" effect they found:

a) If $|\sin \varphi| \ll \varepsilon/k$ the impedances of homogeneous medium and inhomogeneous layer are so different that total reflection takes place in the plane $z = \Delta$. The reflection coefficient V is then completely determined by the properties of the lower boundary.

b) For $|\sin \varphi| \gg \varepsilon/k$ " the absorption of sound in the layer before reaching the uneven surface is very great. As opposed to the preceding case, screening is now caused by the strong absorption of sound in the layer, with reflection at the lower boundary almost totally absent" (Ref. 29, p. 362).

c) In the intermediate region the surface has to be described in more detail. Taking a periodically random surface with correlation function

$$\Phi(\xi) = h^2 \exp(-\xi^2/a^2) \cos(K\xi) \qquad \text{(Eq. 63)}$$

and assuming that $(K/k)^2 \ll 1$, $Ka \gg 1$ they calculated the reflection coefficient V as a function of ε/k for several layer thicknesses Δ. The result is shown in Fig. 16 with $k = 40\ m^{-1}$ ($f = 10$ kHz), $K = 0.63\ m^{-1}$, $h^2 = 0.1\ m^2$ and $\sin \varphi = 0.01$ (i.e. $\varphi \sim 0.6°$). The curves clearly underline the remarks made in (a) and (b).

4.5 "Doppler" and Other Frequency Effects

Many papers deal with surfaces that are independent of time. But a simple observation at sea shows that a realistic description of its surface is not possible without introduction of the time variable.

Because of the time-dependency of the ocean surface the transmission of a monochromatic wave results in a received signal that shows random fluctuations in amplitude and phase, when they are recorded as a function of time (see also Section 3.5.1).

Such a signal suffering from amplitude and phase fluctuations " can be thought of as being a carrier that is simultaneously amplitude and phase modulated by a random signal. Use of the term "modulation" is justified, since the highest significant frequency components of both amplitude and phase are small as compared to the carrier frequency " (Ref. 25, p. 706).

Since the phenomenon is due to movement of the surface elements the terms "Doppler effect" or "frequency smear" are also used.

Rojas (Ref. 71) expressed the surface movement as a change in the length of the propagation path. He assumed that this quantity can be represented by a Gaussian process. In his very straightforward approach the carrier is only phase modulated, again with Gaussian distribution. He calculated the power spectrum of the return signal

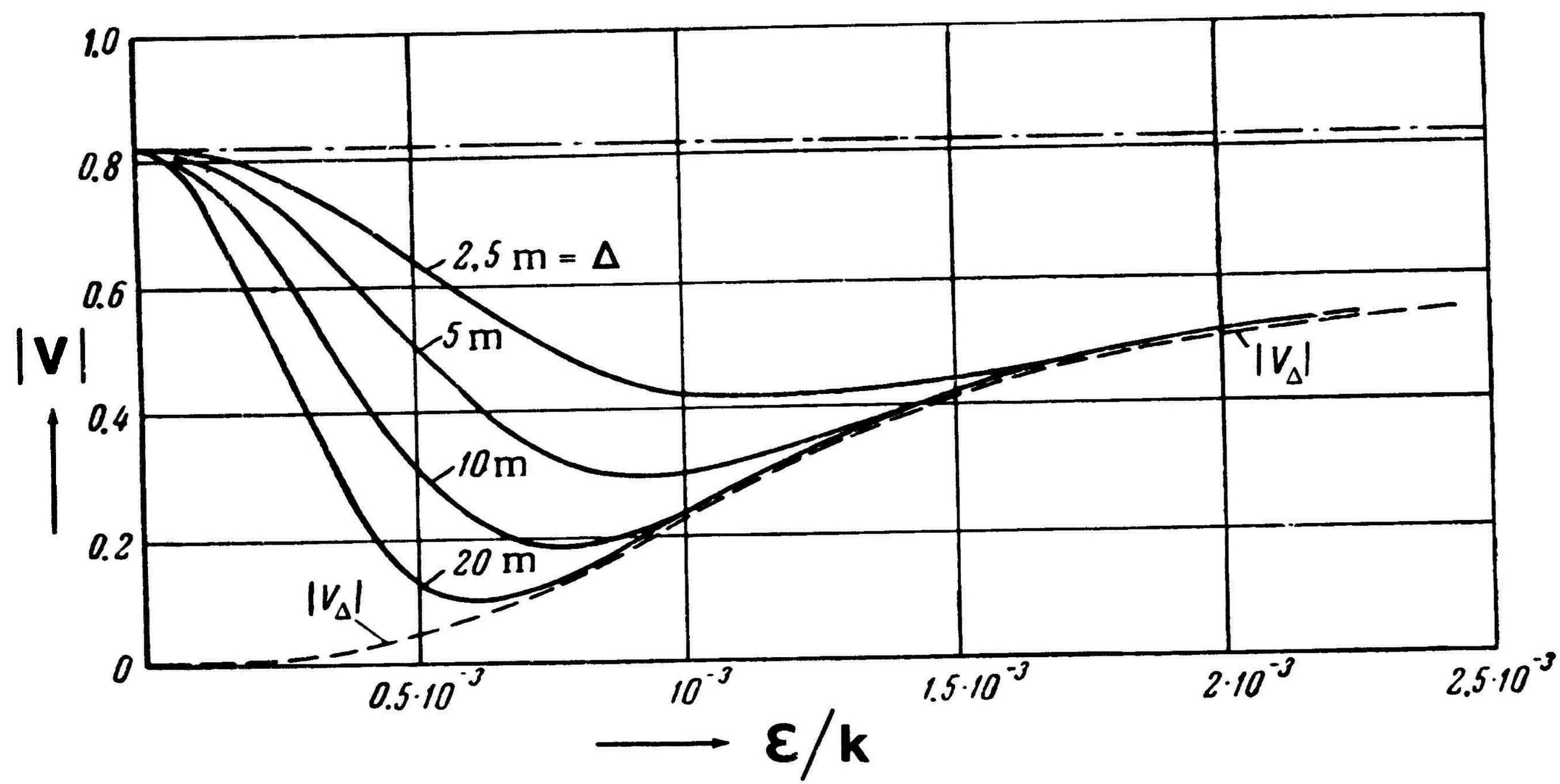

FIG. 16 REFLECTION COEFFICIENT OF A RANDOM SURFACE CORRELATION FUNCTION $\Phi(\xi) = h^2 \exp(-\xi^2/a^2) \cos(K\xi)$, BELOW WHICH A LAYER OF AIR BUBBLES IS PRESENT

ε is the concentration of air bubbles, Δ the layer thickness, f = 10 kHz (i.e. k = 40) K = 0.63 /m, h^2 = 0.1 m^2 and ϕ = 0.6°. The value for a is not reported. (From: Glotov and Lysanov – Ref. 29)

and found it to be Gaussian shaped and centred at the original frequency. His formulae hold only for (near) normal incidence, but the influence of a different geometry could have been incorporated easily, with similar results.

When a sinusoidal surface (wave number K) moving with constant speed v is considered, as has been done by Gulin (Ref. 31), the scattered waves of order m are Doppler shifted over a frequency ω_D that is given by:

$$\omega_D = m K v \qquad \text{(Eq. 64)}$$

It follows from this formula that the specularly reflected wave ($m = 0$) is not influenced by the Doppler effect. This is correct, as the specular reflection comes from the "average" (flat) surface.

This case may seem somewhat theoretical since the ocean surface has a spectrum of sinusoidal waves rather than a single wavelength. But Liebermann stated that "monochromatic radiation will be preferentially scattered according to the familiar diffraction grating formula" (Ref. 43, p. 932): for a given geometry the scattering of a monochromatic wave is mainly produced by the surface wave of length Λ, where

$$\Lambda = \lambda\left(\sin\,\theta_{in} + \sin\,\theta_{out}\right)^{-1}, \qquad \text{(Eq. 65)}$$

i.e. the scattering has a resonant character. This fact is also expressed by Eqs. 19 and 47.

Measurements made by Liebermann (Ref. 43) have confirmed his statement, and formulae derived by Marsh also indicate that "the reverberation spectra will be narrow and centred at frequencies $\omega \pm \omega_D$" (Ref. 53, p. 1836).

Parkins (Ref. 65) recently calculated the spectral density of the waves scattered from a Gaussian surface described by the Neumann-Pierson directional wave spectrum. Two cases have been considered:

the slightly rough surface (low frequency or low sea state) and the very rough surface (high frequency or high sea state). "The reradiation from a surface only slightly rough is found, expectedly, to be principally a reflection; the departure from this has been shown to be caused by the surface roughness of propagating gravity waves. For a very rough surface, there is diffuse scattering that causes the spectral line of the reflection to broaden into a Gaussian curve which shifts and changes in width with sea state and the angles of incidence and observation" (Ref. 65, p. 1267).

4.6 Geometrical Shadowing

The phenomenon of "shadowing" of certain surface areas by other parts of the boundary, which can occur when the surface irregularities are large with respect to the wavelength of the incident radiation and at small grazing angles, has been treated separately. The papers devoted to this phenomenon are concerned with the calculation of a "shadowing function", based on the statistics of the surface, with which the scattering area has to be weighted. No papers have been found in which the shadowing function is applied.*

The starting point in this area of investigation is the article by Beckmann (Ref. 14). His method, extended by others, can be explained with the aid of Fig. 17, in which a plane monochromatic wave is

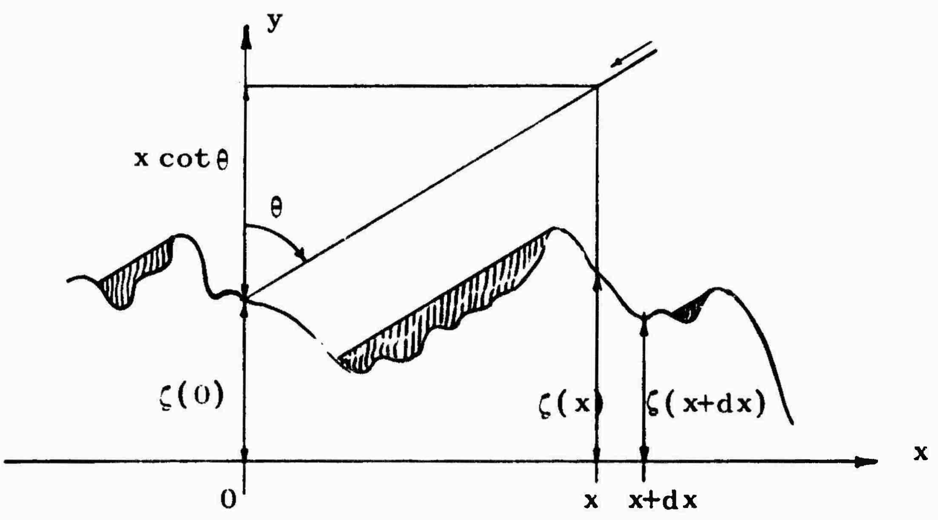

FIG. 17 SHADOWING OF A RANDOM ROUGH SURFACE (From Beckmann – Ref. 14)

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*A possible exception is Ref. 39 because the modified boundary condition (Eq. 48) can be interpreted as introducing a shadowing function with the value 0.5.

incident on a rough surface with incident angle θ. The shadowing function S is the probability that the point $\zeta(0)$ is illuminated. If we let

$$s(\theta, x) = \text{Prob}\left[\zeta(0) \text{ is not shaded by any } \zeta \text{ up to } \zeta(x)\right] \qquad \text{(Eq. 66)}$$

the required shadowing function is

$$S(\theta) = \lim_{x \to \infty} s(\theta, x). \qquad \text{(Eq. 67)}$$

A differential equation for $s(\theta, x)$ can be derived and solved, via the infinitesimal interval $(x, x + dx)$.

For the limit of Eq. 67 Beckmann found

$$S(\theta) = \exp\left[- \int_0^\infty q(x)\, dx\right] \qquad \text{(Eq. 68)}$$

where $q(x)\, dx$ is the probability that $\zeta(0)$ is shaded by ζ in the interval $(x, x + dx)$, given it is not shaded by ζ in $(0, x)$. This probability is put approximately equal to the probability that ζ will interrupt the ray directed towards $\zeta(0)$ in $(x, x + dx)$ with slope greater than that of the ray, i.e. cot θ. Hence, the integrand in Eq. 68 contains two conditions: one on the surface elevation ζ in $(x, x + dx)$ and one on the slope ζ'. Although these quantities are correlated, Beckmann treated them as independent "so as not to complicate matters". The resulting error "turns out to be zero for symmetrical distributions" (Ref. 14, p. 385). The general solution is therefore

$$S(\theta) = \exp\left[-\tfrac{1}{2} \tan \theta.\, \text{Prob}\left(\zeta' > \cot \theta\right)\right], \qquad \text{(Eq. 69)}$$

from which special cases can be studied. For a surface with Gaussian

correlation function Eq. 69 yields

$$S(\theta) = \exp\left[-\tfrac{1}{4} \tan \theta \cdot \operatorname{erfc}\left(\frac{a \cot \theta}{2h}\right)\right]. \tag{Eq. 70}$$

It is important to note that in Beckmann's calculation of $S(\theta)$ only the elevation $\zeta(0)$ of the surface observation point has been considered. But the slope $\zeta'(0)$ also plays a role: if its value is larger than $\cot \theta$ the point will certainly be shaded. This fact has been recognized by Wagner (Ref. 80). He calculated $S(\theta)$ for given $\zeta(0)$ and $\zeta'(0)$, using Beckmann's method. He found instead of Eq. 68):

$$S\left(\theta \mid \zeta(0), \zeta'(0)\right) = \exp\left[-\int_0^\infty q(x)\, dx\right] U\left(\cot \theta - \zeta'(0)\right) \tag{Eq. 71}$$

where U is the unit step function. To obtain $S(\theta)$, Eq. 71 has to be averaged over all possible values of height and slope. Wagner performed this operation while maintaining the correlation between these quantities.

A simplified method for the evaluation of the integral in Eq. 59 has been published by Smith (Ref. 75). He neglected the correlation between height and slope, but obtained for Gaussian Φ results that do not differ significantly from the more complete solution of Wagner (see Fig. 18).

Shadowing in the case of backscattering has been simulated on a digital computer by Brockelman and Hagfors (Ref. 16). Their shadowing function $R(\theta)$ puts special emphasis on those surface elements that are perpendicular to the line of sight of the observer. This different concept of shadowing, which is based on reflecting facets, caused serious disagreement with Beckmann. (See Ref. 16, p. 626: Discussion.)

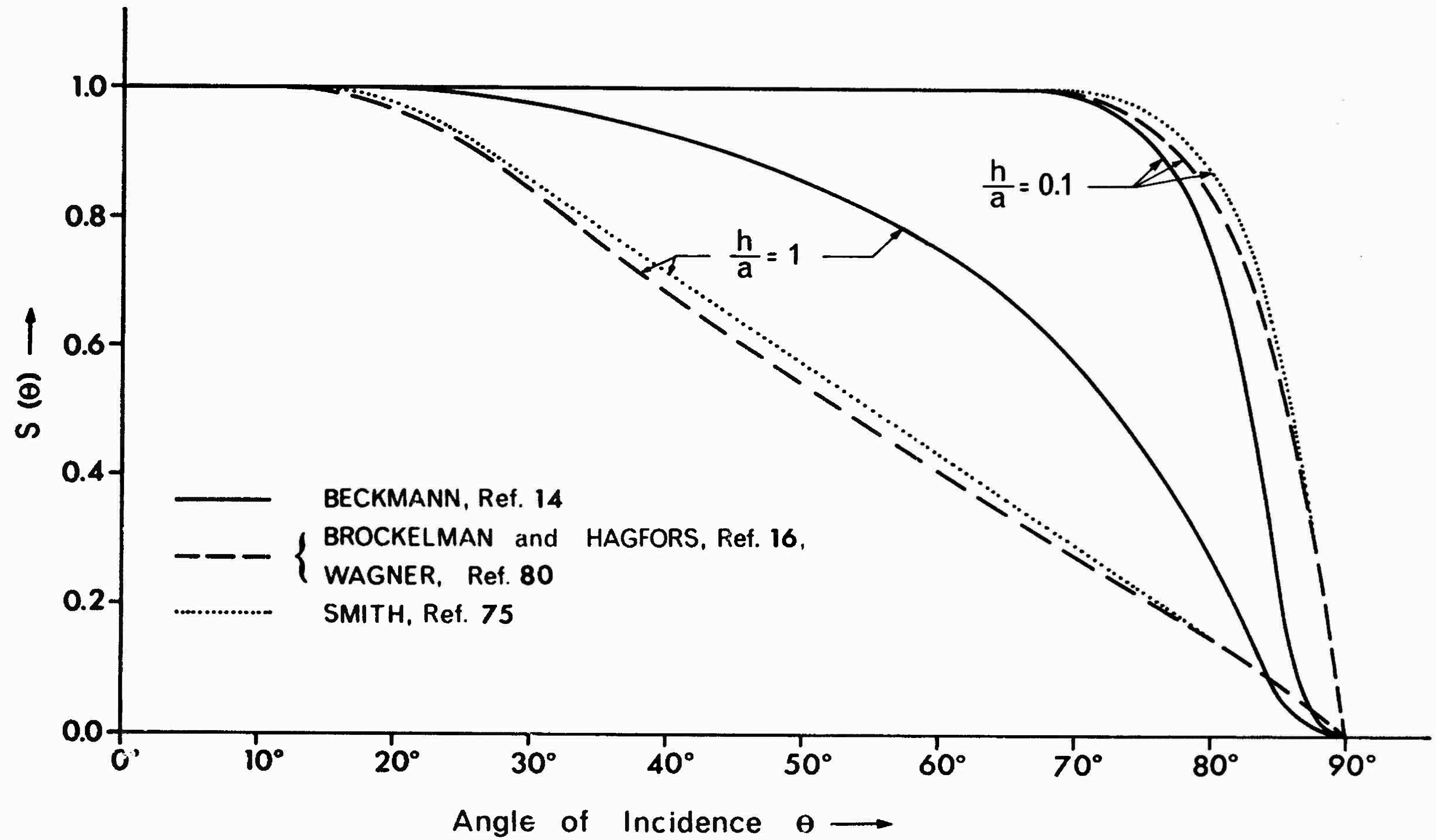

FIG. 18 THE SHADOWING FUNCTION $S(\theta)$ FOR A SURFACE WITH GAUSSIAN CORRELATION FUNCTION $\Phi(\xi) = h^2 \exp(-\xi^2/a^2)$; $S(\theta)$ is the probability that an arbitrary surface point is illuminated by a plane wave with angle of incidence θ.

Brockelman and Hagfors generated on a digital computer a stationary "noise" with Gaussian correlation function. For a set of N samples they calculated for a given θ:

N_s the number of shaded samples

N_R the number of reflection points

N_{Rs} the number of shaded reflection points.

In digital form Beckmann's shadowing function becomes

$$S(\theta) = (N - N_s)/N \tag{Eq. 72}$$

whereas Brockelman and Hagfors used

$$R(\theta) = (N_R - N_{Rs})/N_R. \tag{Eq. 73}$$

Even the comparison of curves according to Eqs. 70 and 72 for Gaussian Φ led to disagreement (see Fig. 18). But, as Brockelman and Hagfors remark (Ref. 16, p. 625), this discrepancy may possibly be explained by a mathematical error in Beckmann's paper (Ref. 14). This error was first noted by Shaw (Ref. 73).

In Fig. 18 we have combined some results of the papers mentioned. The disagreement between Beckmann's theory and the computer "experiment" is especially large for the rougher surface (h/a = 1). It is also clear that Wagner is in excellent agreement and that the simplified approach of Smith is very useful.

4.7 The Inverse Problem

In the preceding parts of this study it has become clear that there is a strong relation between the characteristics of the scattered field and the surface correlation function $\Phi(\xi, \eta)$.

In most cases certain assumptions are made about this function, to enable the continuation of the analysis. In particular the expressions for Φ as given in Eqs. 55 and 56 are often met, but the more realistic theory of surface wave spectra is gaining ground (see Section 4.8.2).

The present section deals with the opposite procedure, i.e. the problem how Φ and related parameters can be derived from the properties of the secondary field.

Eckart (Ref. 26) was the first to touch upon this "inverse problem". He observed that the surface wave spectrum F (the Fourier transform of Φ) could in theory be measured for low frequencies via σ (see Eq. 47). "Unfortunately, it is necessary to vary the directional parameters α and β as well as the frequency of the incident radiation. This may be difficult in practice" (Ref. 26, p. 568). Even more disappointing is the result for high frequencies: in that case σ does not contain the function Φ but only the variances of the slopes.

Proud, Beyer and Tamarkin (Ref. 69), who have slightly modified the Eckart theory, expressed $\Phi(\xi)$ as the ratio of two empirical functions: one is related to the scattered intensity as a function of frequency; the other describes the source radiation pattern. The formula holds for a smooth surface: $|k\zeta_{max} \cos \theta| << 1$. The authors showed "that in theory one can form an estimate of the reflecting surface correlation function from acoustic measurements alone. It was shown, furthermore, that all the information about the surface is contained in the backscattering" (Ref. 69, p. 552).

A very simple experiment to perform a spectral analysis of a rough surface has been described by Liebermann (Ref. 43). He used the fact that the scattering of a monochromatic wave from a rough surface is resonant: for a given geometry and incident wavelength it is mainly a narrow band of surface waves that produces the scattering. Hence, "a 'spectrum' analysis of surface roughness can be obtained by slowly varying the frequency of the incident monochromatic radiation and observing the magnitude of the scattered radiation" (Ref. 43, p. 932). Marsh (Ref. 53) provided the corresponding formulae for

the two-dimensional case and showed how the reverberation spectrum and the surface wave spectrum are related.

Medwin (Ref. 56) analyzed the specular reflection from a wind-driven surface at normal incidence and for several values of the roughness parameter χ, as defined by Eq. 54. He found that measurement of the specularly reflected intensity makes it possible to predict the rms wave height if $\chi^2 \leq 0.1$, and the rms surface slope if $\chi^2 \geq 10$.

4.8 The Surface of the Ocean

4.8.1 Surface Height and Slopes

In all studies that deal with a random surface it is assumed that the surface elevation and slopes can be considered as Gaussian processes, stationary (in time) and homogeneous (in space). It has become clear from measurements that this assumption, although made mainly for computational reasons, is fortunately not too far from reality.

Kinsman recorded wave height with a capacitance pole and computed the probability density function of the surface displacement from 24 records (Ref. 4, p. 345). He found curves close to normal, as is illustrated in Fig. 19. As for the surface slopes, these have been studied by Cox and Munk (Ref. 24). Their method consisted "in photographing from a plane the sun's glitter pattern on the sea surface, and translating the statistics of the glitter into the statistics of the slope distribution. Winds were measured from a vessel at the time and place the photographs were taken. They ranged from 1 m/s to 14 m/s.

"If the sea surface were absolutely calm, a single, mirror-like reflection of the sun would be seen at the horizontal specular point. In the usual case there are thousands of 'dancing' highlights. At each highlight there must be a water facet, possibly quite small, which is so inclined as to reflect an incoming ray from the sun towards the observer. The farther the highlighted facet is from the horizontal specular point, the larger must be this

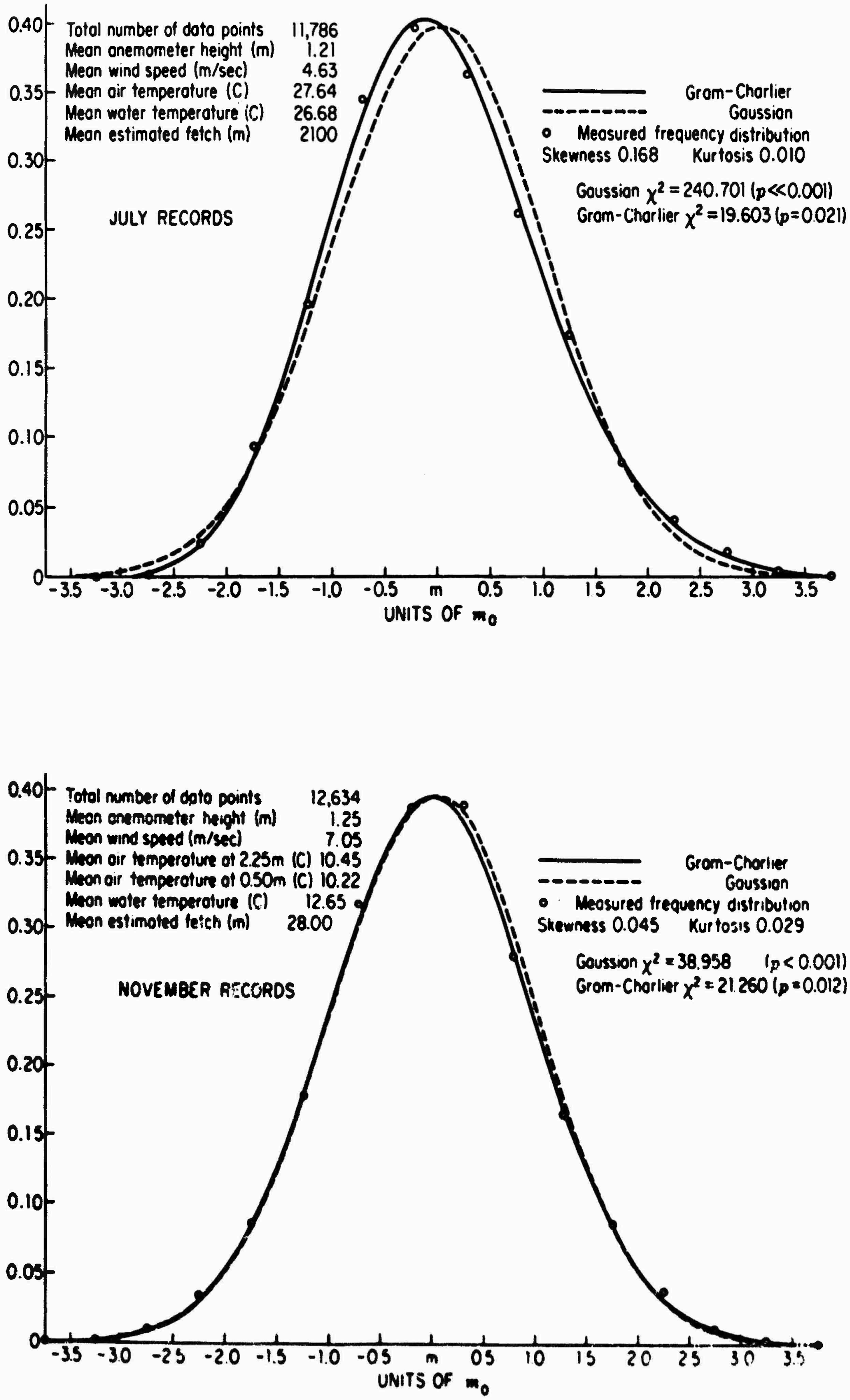

FIG. 19 DISTRIBUTIONS OF WATER SURFACE DISPLACEMENT
(From Kinsman – Ref. 4)

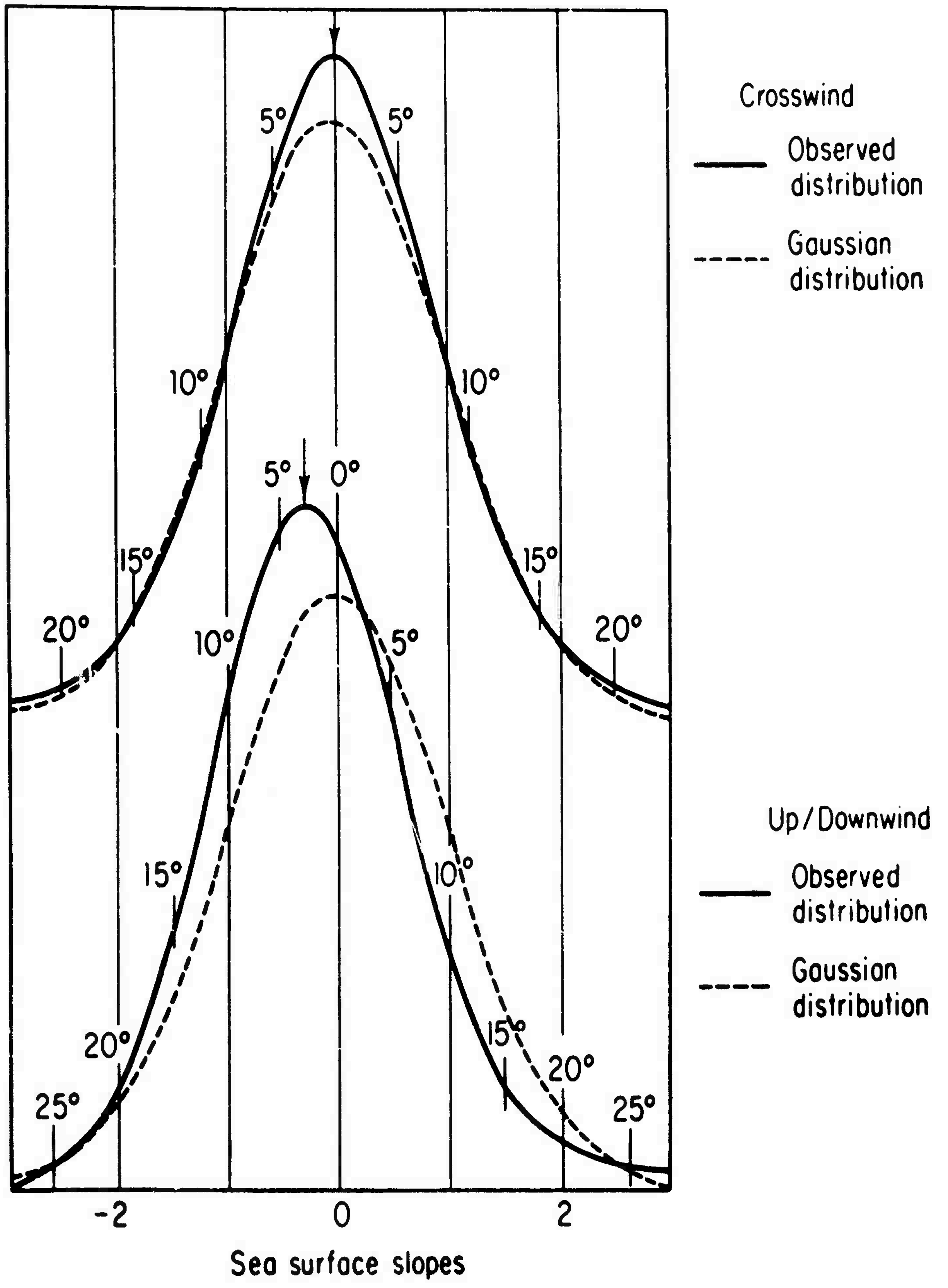

FIG. 20 DISTRIBUTIONS OF SEA SURFACE SLOPES
(From: Cox and Munk – Ref. 24)

inclination. The width of the glitter pattern is therefore an indication of the maximum slope of the sea surface" (Ref. 24, p. 838).

As can be seen from Fig. 20, the slope density functions also are not far from a normal curve. This is "appropriate to a wide, continuous band spectrum but not to a few discrete frequencies" (Ref. 4, p. 350).

4.8.2 The Surface Correlation Function and Wave Spectrum

As for the correlation function of the surface roughness, mainly two types have been applied, namely the ones given in Eqs. 55 and 56. They have been chosen for their relative simplicity in the evaluation of integrals. Moreover, the first one is not too bad for "swell", a narrow band type of waves.

More realistic, however, seems the introduction of the theory of a surface wave spectrum, which is very well explained by Kinsman (Ref. 4), among others. In this theory the surface roughness is considered as the combined effect of a band of surface waves that travel in all directions over the surface, each of them having its own wavelength. For the deep ocean the waves are gravity waves: their wave number K is related to the frequency ω_s via

$$K = \omega_s^2 / g . \qquad \text{(Eq. 74)}$$

In general the surface correlation function Φ depends on the position of the points of observation and on the observation times. But for a stationary and homogeneous surface it is only the differences in position and in time that count. Then the surface wave spectrum, which is the Fourier transform of Φ, becomes (in vector notation):

$$F(\vec{K}, \omega_s) = (2\pi)^{-3} \iint d\vec{\rho} \int d\tau \; \Phi(\vec{\rho}, \tau) \cdot \exp\left[-i(\vec{K}\cdot\vec{\rho} - \omega_s \tau)\right] . \qquad \text{(Eq. 75)}$$

This function $F(\vec{K}, \omega_s)$ is still too general for practical use. Easier to handle is the energy spectrum function $A^2(\omega_s, \alpha)$, where α represents the direction of travel of the wave with frequency ω_s. It reduces to $A^2(\omega_s)$ when the surface is isotropic. Then $\Phi(\vec{\rho})$ becomes simply $\Phi(\rho)$.

The relation between $A^2(\omega_s)$ and $\Phi(\rho)$ can easily be found. A plane wave with frequency ω_s and direction α arrives at two observation points, situated on the X-axis at distance ρ, at times that differ by an amount τ, such that

$$\tau = \frac{\rho \cos \alpha}{u}. \qquad \text{(Eq. 76)}$$

Here u is the frequency-dependent wave velocity:

$$u = g/\omega_s, \qquad \text{(Eq. 77)}$$

which follows from Eq. 74. The contribution of this wave to $\Phi(\rho)$ equals $e^{-i\omega_s \tau}$; this has to be averaged over all possible directions and weighted with the energy spectrum $A^2(\omega_s)$. The final result is

$$\Phi(\rho) = \tfrac{1}{2}\int_0^\infty d\omega_s \; A^2(\omega_s) \; J_0(\omega_s^2 \rho/g), \qquad \text{(Eq. 78)}$$

a formula applied by Marsh (Refs. 49, 50, 51).

There is some disagreement in the literature about the explicit form of the function $A^2(\omega)$. At least part of the discrepancies can be explained by realizing that the measurements on which the empirical formulae for $A^2(\omega_s)$ are based have not all been made in seas with the same state of development. We have already observed that this fact also plays a role in the different outcomes for backscattering measurements (see Section 3.5.2).

When a constant wind starts creating waves on the sea surface, the stationary situation (that is, a "fully aroused sea") is not reached immediately but after a certain lapse of time. Before that moment the sea is partially developed and has a wave-spectrum that is

different from that of the completely developed sea. When the wind stops, or when the waves travel outside the "fetch" where they have been generated, their spectrum changes from broadband ("sea") to a narrow band and low frequency spectrum ("swell"), because the low frequencies outrun the high ones (cf Eq. 77). An excellent account of the generation and propagation of ocean waves is given by Kinsman (Ref. 4).

Marsh (Refs. 49, 50) applied the Neumann-Pierson model for $A^2(\omega_s)$, in which the wind speed v appears as a parameter:

$$A^2(\omega_s) = C\omega_s^{-6} \exp(-2g^2/\omega_s^2 v^2); \qquad \text{(Eq. 79)}$$

v is expressed in cm/s and $C = 4.8 \times 10^4$ cm²/s⁵. Parkins (Ref. 65) used the anisotropic version

$$A^2(\omega_s, \alpha) = C\omega_s^{-6} \exp(-2g^2/\omega_s^2 v^2) \cos^2 \alpha \qquad (-\pi/2 \le \alpha \le \pi/2), \qquad \text{(Eq. 80)}$$

and $C = 3.05$ m²/s⁵. In a more recent paper Marsh stated: "Arguments have been presented that a more satisfactory form of the equation is

$$A^2(\omega_s) = C\, g^2\, \omega_s^{-5}, \qquad \text{(Eq. 81)}$$

where $C = 7.4 \times 10^{-3}$, an absolute, dimensionless constant". This formula "contains no dependence on wind speed and is intended to apply to the fully developed sea" (Ref. 51, p. 240).

The meaning of "fully developed sea" or "fully aroused sea" can be understood with the function $A^2(\omega_s)$. It is a sea whose spectrum, for a given wind speed, contains components of all frequencies $0 \le \omega_s < \infty$, each with the maximum energy of which it is capable under the given wind.

The total energy in a fully aroused "Neumann sea" can be found by integration of Eq. 79 over ω_s from 0 to ∞ (Ref. 4, p. 390). With Eq. 78 it can be seen that this integral equals $2\,\Phi(0)$, or $2h^2$.

Finally we note another interesting feature of the Eckart theory: it contains (at least for low frequencies) the surface wave spectrum $F(\vec{K})$ and is therefore not restricted to the Gaussian function chosen by Eckart.

With the anisotropic version of Eq. 78, i.e.

$$\Phi(\vec{\rho}) = \frac{1}{2}\int_{-\pi}^{\pi} d\alpha \int_{0}^{\infty} d\omega_s \quad A^2(\omega_s, \alpha)\, \exp\left[-i\omega_s^2\, \rho \cos \alpha/g\right], \qquad \text{(Eq. 82)}$$

the theory of surface wave spectra can be used in Eq. 47, because $F(\vec{K})$ is the Fourier transform of $\Phi(\vec{\rho})$:

$$F(\vec{K}) = (2\pi)^{-2} \iint d\vec{\rho}\ \Phi(\vec{\rho})\, \exp(-i\, \vec{K}\cdot\vec{\rho}). \qquad \text{(Eq. 83)}$$

CONCLUSION

1. Scattering and reflection of sound waves by the sea surface is dependent on time, on frequency of incident waves, and on the geometry of transmitter and receiver. No theoretical models have been found in which these three basic variables are considered simultaneously, except the quasi-phenomenological model (Middleton). This latter model, however, has a serious disadvantage: it is based on quantities that have to be found by experiment.

2. Almost all scattering theories are only valid for smooth surfaces (small slopes). Of these theories the Eckart approach has been applied most frequently, because of its relative simplicity. The Rayleigh procedure, and its generalization for random boundaries (Marsh), is based on a seriously criticized assumption. For very smooth surfaces, however, its results are comparable with those of other theories.

3. The Uretsky theory not only covers the scattering at smooth boundaries, but also gives a fairly good prediction for rough boundaries. Unfortunately it has been developed only for a sinusoidal surface.

4. The surface elevation and slopes are generally assumed to be stationary, Gaussian processes. Measurements at sea have indeed shown the validity (with limitations) of this assumption.

5. The most realistic way to incorporate the correlation functions of surface height and slopes is via the theory of the surface wave spectrum (Neumann-Pierson).

6. The scattering of a monochromatic wave at a random surface is resonant: the scattering is mainly produced by a small band of surface waves that fit the incident radiation (Liebermann).

7. The backscattering contains all statistical information about the surface. An acoustical determination of the surface statistics is therefore possible, in theory.

8. A large quantity of experimental data has been collected at sea and by using model tanks. The influence of several parameters has been studied: wind velocity, frequency of incident radiation, grazing angle etc. The data indicate three mechanisms: reflection by wave-facets near normal incidence, scattering by small air bubbles below the surface at small grazing angles, and scattering by irregularities that are small compared with the incident wavelength in the intermediate region (Urick).

9. No correlation has been found between the height of a random surface and the reflection coefficient. The second order statistical moments of the diffracted field (spatial correlation functions, intensity, etc.), however, show good correlation with the surface irregularities.

ACKNOWLEDGMENT

The study of the papers by Marsh et al. (Refs. 48, 49, 50, 51) has been facilitated by the assistance of Mr. Gerhard Wittek from the "Max-Planck-Institut für Strömungsforschung", Göttingen, W. Germany, who worked at SACLANTCEN during the summer of 1968.

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Abbreviations for journals appearing more than once:

AP	=	Antennas and Propagation (IEEE Trans.)
IT	=	Information Theory (IEEE Trans.)
JAP	=	Journal of Applied Physics
JASA	=	Journal of the Acoustical Society of America
JOSA	=	Journal of the Optical Society of America
SP-A	=	Soviet Physics — Acoustics (English translation)
(L)		Letter to the Editors
(C)		Communication

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